A numerical study of open atmospheric balloon dynamics

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The dynamics of open balloons in an atmosphere may be studied with a body-fluid coupled model. A numerical approach is required to solve the corresponding equation set. Solutions under different conditions are obtained here for the vertical and one horizontal direction. Relevant dynamical features during ascent, flotation, and descent depend on balloon thermodynamics, wind, air small-scale turbulence, and perturbations to the background atmosphere. After analysis of the results it is found that approximate analytical solutions may be found in certain cases. The effect of nonlinear drag on balloon oscillation period and damping near flotation is evaluated. © *2003 American Institute of Physics.* [DOI: 10.1063/1.1607325]

I. INTRODUCTION

Balloons have been increasingly attracting the interest of researchers. They have been used as scientific platforms with useful loads for experiments, space observations, and particularly for the study of the atmosphere of the Earth and other planets. The main atmospheric applications are for the study of phenomena and for *in situ* measurements. A relevant advantage is given by the unimprovable spatial and temporal resolutions of collected data, considering the moderate balloon speeds and the high performance of actual instruments. The highest vertical resolution for atmospheric observations, on the order of 1 m, stems from balloons.[1] Their mission success is rated among many factors by the payload weight they are able to lift, the operational altitude range, and the duration at target height with minimum excursions.

Balloons fall into two basic categories: open (zero-pressure) and closed (superpressure). The former have one or several openings at the base of the envelope and are therefore said to be open to the external atmosphere. The film is either filled with hot air or a gas lighter than the atmosphere. These balloons usually fly science and technology missions up to the stratosphere, lasting a few hours to a few days. They vary in volume from several thousand to over one million cubic meters and can carry payloads weighing tens of kilograms to a few tons. Their altitude can be controlled by a valve at the top of the balloon and by ballasting.

A balloon responds to its environment in a complex way. Essentially, its horizontal speed mimics the ambient air, but even this simple assumption may be sometimes questioned. Its vertical motion is a more elaborate function of its own characteristics and the surroundings due to the additional incidence of buoyancy and gravity. Some types of closed balloons have been used as horizontal quasi-Lagrangian tracers of air parcels (see, e.g., a work by Morel and Bandeen[2]), and vertical air motions have been said to be inferred from changes in the ascent rate in some cases (see, e.g., a study by Kitchen and Shutts[3]). On the other hand, it has been observed that open balloons may follow density variations rather than vertical motions of air.[4] In the last decades a considerable number of studies have been reported in relation to the response of closed balloons to atmospheric variations and a few can be mentioned.[5–7] The behavior of open balloons has also received some attention (we may cite, for example, two works[8,9]) and many issues remain open. These objects represent a significant challenge as they are specifically subject to unknowns related to their changing size (e.g., the determination *a priori* of the flotation altitude), which is strongly dependent on gas amount and thermodynamics during flight. A need for an adequate understanding of open balloon dynamics in the atmosphere and their general behavior has emerged. It becomes desirable to develop skills to predict the balloon equilibrium altitude, the natural frequency of vertical oscillation around it (as in any stratified fluid), the response to the mean, and regular or turbulent fluctuations in the environment or the general behavior under different conditions. The dynamical response represents by itself an essential feature to be explored in detail, as it can give us information about air or balloon parameters. In particular, any thorough analysis of balloon response as a function of atmospheric parameters could be used to derive the inverse relation.

Balloon studies often had significant restrictions for the sake of analytical solutions or just for simplicity. Only the vertical motion was considered, or the analysis was just performed at flotation level, or exclusively the forces due to weight, upthrust, and drag were included, or no thermodynamics was contemplated. A thorough analysis of the problem needs to remove or loosen these restrictions.[10,11] However, no analytical solutions may be found then, apparently. Below, we perform a study using a body-fluid coupled model, implemented in a computational program[11] which finds the dynamics in the vertical and one horizontal direction of an open balloon in an atmosphere. The environment and gas thermodynamics are taken into account. Our numerical tool is able to simulate flights of a few hours up to the stratosphere in terms of air and balloon parameters. The code may be used for ascent, descent, and flotation. A wide range of atmospheric and balloon conditions can be represented.

The program is capable, with little adaptation, of simulating flights of open balloons of a range of sizes, made of skins with diverse thermal characteristics, that use various lifting gases, employ an exhaust valve and ballast, and experience different environments (including non-Earth atmospheres) under changing conditions.

The aim of the present work is to gain a qualitative understanding of open balloon dynamics in the Earth atmosphere. In particular, we wish to study the behavior (e.g., lag and amplitude of response as a function of balloon and atmosphere properties) under forcing by a mean and regular or turbulent fluctuations in the environment. A description of the balloon model and its implementation in a numerical program is given below. In the following section, a systematic study through a variety of simulations and a detailed analysis of them may be found. Conclusions will be drawn in the last part.

II. THE BALLOON MODEL AND PROGRAM

A set formed by coupled equations describing the behavior of an open balloon and the atmosphere in which it is immersed has already been presented.[11] It is assumed that the balloon basically resembles a sphere regardless of the volume of the gas bubble inside it. The air and gas pressures are assumed to be equal at the bottom opening. Skin friction drag, aerodynamic lift, and azimuthal rotation or pendulous motion may be neglected. The governing equation for the motion of the balloon stems from the equation for the conservation of momentum of a body coupled with the equation for conservation of momentum of a fluid. For large Reynolds number (Re) it becomes

$$\begin{aligned} M_b\frac{d\mathbf{v}_b}{dt}+\frac{1}{2}M_a\frac{d\mathbf{v}_b}{dt}=&\frac{1}{2}M_a\frac{D\mathbf{v}_a}{Dt}-\frac{1}{2}C_DA\rho_a|\mathbf{v}_b-\mathbf{v}_a|(\mathbf{v}_b\\ &-\mathbf{v}_a)+M_a\frac{D\mathbf{v}_a}{Dt}+(M_b-M_a)\mathbf{g}\\ &-\frac{1}{2}M_a(\mathbf{v}_b(0)-\mathbf{v}_a(0,\mathbf{X}(0)))\,\delta(t)\\ &-\dot{M}_b\mathbf{v}_g, \end{aligned} \tag{1}$$

where $\mathbf{v}$, ρ, and M, respectively, correspond to velocity, density, and mass, r is the radius of the sphere, μ the fluid dynamic viscosity, $\mathbf{g}$ represents gravity, t gives time, A is the effective cross-sectional area of the sphere, and C_D is the drag coefficient, which is a function of $\mathrm{Re}=2r|\mathbf{v}_b-\mathbf{v}_a|\rho_a/\mu$, while subindices $_a$, $_b$, and $_g$ refer hereinafter to the air, balloon, and gas. Notice that $\mathbf{v}_a$, ρ_a, and μ are fluid quantities evaluated at the current position of the body $\mathbf{X}(t)$. The mass of air displaced by the whole system is

$$M_a=\rho_aV_a, \tag{2}$$

where V_a is the corresponding volume of air (except for the little space occupied by the skin, gondola, and payload, $V_a=V_b=V_g\equiv V$), whereas M_b is the total mass (gas, ballast, skin, gondola, payload) and $\dot{M}_b$ refers to the time rate of mass being expelled with $\mathbf{v}_g$, the gas release velocity relative to the balloon (it is nonzero for gas expelled through an apex valve). It should also be remarked that D/Dt and d/dt are, respectively, the total derivatives following a fluid element and the body. The origin of the terms of the equation of motion may be found in various publications.[12–15] On the left-hand side we find the net force on the balloon and the component of the inertial drag force due to the balloon. The terms on the right-hand side, respectively, correspond to the inertial drag due to the fluid, steady state (also called aerodynamic or form) drag, dynamic buoyancy (due to pressure gradient), body force (weight plus upthrust), a correction due to an initial differential velocity between fluid and body, and a kind of retropropulsion effect. The expression for the inertial added mass terms is valid for large balloon frequencies ω_b, which should be interpreted as $(\omega_b r^2\rho_a/\mu)^{1/2}\gg 1$ (see the books by Landau and Lifshitz, and Batchelor[12,14]). The equation of motion is clearly nonlinear. Regarding the mentioned ranges of validity, it should be remarked that in the simulations to be found below the conditions $10<\mathrm{Re}$ and $10<(\omega_b r^2\rho_a/\mu)^{1/2}$ hold.

Solutions require the specification of the air variables. A representative atmosphere with small-scale perturbations and a prominent gravity wave is included in the model. The background air temperature in the height range 0–32 km and the density at sea level are taken from the US Standard Atmosphere.[16] The atmospheric density and pressure profiles with altitude then become uniquely defined. Dynamic viscosity is approximately constant ($1.50\times10^{-5}\ \mathrm{N\,s\,m^{-2}}$). The mean flow field profile is null in the vertical direction, whereas the horizontal component is specified by linear shears in three neighboring height ranges. The perturbations to the mean atmospheric background state have been separated into two ranges: one in which the typical lengths are larger than the balloon diameter and dominated by the propagation of gravity waves, and another where the small-scale effect of velocity may be parametrized by a turbulent viscosity μ_T, which increments the effective value to $\mu+\mu_T$. The linear theory by Hines[17] predicts that the relative phases and amplitudes of velocity, pressure, and density fluctuations in a gravity wave are not arbitrary, but are given by so-called polarization relations. At the present time no balloon studies seem to have been made which accommodate simultaneous variations of atmospheric density, temperature, and vertical and horizontal velocity components due to a gravity wave. We represent these fluctuations as pertaining to a time-dependent, bidimensional, and monochromatic gravity wave, and convective saturation may be included. A more complete scenario would include a broad power spectrum, but just a single large monochromatic wave has been included because data up to the lower stratosphere[18,19] indicate that usually one energy-dominant mode with a vertical wavelength of a few km is likely to be present. Any estimate for the small-scale viscosity should be bounded between the molecular and the large-scale turbulent values, i.e., between 10^{-5} and $10^{5}\ \mathrm{N\,s\,m^{-2}}$.[20]

The balloon equation of motion will also depend on the descriptions for C_D and M_a. The former will be given as[21]

$$C_D=\begin{cases}0.5+24/(\mathrm{Re}+0.01) & \text{for } \mathrm{Re}\leqslant 4.5\times10^5\\ 0.3 & \text{for } \mathrm{Re}>4.5\times10^5,\end{cases} \tag{3}$$

whereas the latter by Eq. (2) depends on V. Balloon volume increases or decreases as the environmental conditions change due to their natural variation or due to the displacement. The gas usually alters temperature at its own rate due to these expansions or contractions, and there is an additional reluctance to thermalize with the surrounding air (i.e., to equal its temperature), so a gradient between both exists and a heat exchange develops. This in turn leads to further volume changes, implying that the mass of displaced air or upthrust and therefore the force balance are modified. The behavior of an open atmospheric balloon is then given, among other parameters, by the heat transfer budget with the environment. Convection, conduction, and radiation may all be included in a study with different degrees of significance. The radiation ambient depends on altitude, cloud cover, and some atmospheric conditions. It mainly includes two bands: infrared (due to the Earth surface and atmosphere) and visible (direct from the sun or reflected). The input and output to the balloon system is one of the significant influence factors on gas temperature. Unfortunately quantitative predictions are subject to some uncertainties that depend on several issues which are difficult to specify,[22,23] so a simple scheme will be applied below. An accurate analytical model of these effects would be extremely complex, if even possible. We will consider the open atmospheric balloon as a thermodynamic system with an ideal gas that exchanges heat with a thermal source (the surroundings). Scenarios where the gas follows the air temperature or undergoes an adiabatic evolution, respectively, imply an infinite or zero rate of heat transfer. Under this scheme, any real situation must be somewhere between these two extremes. With this basic approach there is no need for a detailed heat transfer modeling or for a description of the radiative environment. However, there is obviously no possibility to explore the consequences of the latter on the balloon. Our aim will be to study certain thermal conditions that may bound some characteristics of flights, so we perform a simplified analysis that delimits some ranges of possible observed behaviors. If p and T are, respectively, pressure and temperature, R is the universal gas constant, $R_g = R/W_g$ and W is the molecular weight, the perfect gas state equation

$$p_g = R_g \rho_g T_g, \tag{4}$$

shows that the volume evolution in an open balloon flight will essentially be given by the mass of gas, the air pressure, and the transport of heat through the skin. Then, the equation of motion for the balloon will depend on the gas thermal description through M_a. It should be noticed that previous studies have sometimes not considered the thermodynamic properties. In the present work two diverse scenarios may be modeled: either the gas thermalizes with the surrounding air or it follows a polytropic evolution

$$\frac{p_g}{\rho_g^{\gamma_g}} = c, \tag{5}$$

where c is a constant. This gas law covers the whole spectrum between isothermal and adiabatic behavior if the polytropic index γ_g is contained in the interval between 1and the ratio of specific heats at constant pressure and volume (1.42 and 1.67 for H_2 and H_e, respectively). If appropriate conditions are chosen, then a value of 1 may approximately (a polytropic gas temperature cannot exactly mimic the air because a gravity wave will not induce any fluctuations on it[11]) yield the thermalized case because the gas will follow the air mean temperature if both initial values coincide and if the atmospheric temperature profile is uniform.

Dropping ballast and valving gas are the primary means of flight control for any stage. It may be used to change the vertical velocity or position. Simple applications of ballasting and gas exhaust by an apex valve have been included in an automatic mode in the program. These aspects have been excluded from the present study.

The set that constitutes the full mathematical model includes the balloon equation of motion with expressions for M_a, C_D, ρ_g, V_g, r, A, $\dot{M}_b$, v_g, modified initial conditions if there is a differential velocity between balloon and atmosphere at start, the atmospheric profiles of air temperature, density, and velocity (the molecular and turbulent viscosities are constant and are, respectively, determined by the program and the user), the constraints that prevent strong departure from atmospheric incompressibility (one of the assumptions in the derivation of the balloon equation of motion), the polarization and dispersion relations for gravity waves including if required the saturation condition, and finally the equations related to gas and ballast content.

In an earlier study[24] it was shown that due to the proximity with the helium balloon natural frequency, buoyancy oscillations can be excited by atmospheric waves at the Väisälä–Brunt value (period around 5 min), particularly from 12 000 to 24 000 m. It should be noted that our gravity wave description loses validity when approaching that frequency, so we have chosen to study the response at larger values. In another previous work,[25] 1D analytical solutions were found for vertical balloon oscillations during ascent or descent under the presence of a long period gravity wave for two diverse cases: either the gas thermalized with the surrounding air or it underwent an adiabatic evolution. An isothermal atmosphere, a constant wave velocity amplitude with height, a uniform mean ascent or descent vertical speed of the balloon, a constant drag coefficient, and no gas mass loss during the whole journey were assumed. Most restrictions were necessary to yield analytical results. Consistently with the observations by de la Torre *et al.*,[4] this work found that open balloons where the gas departs from the thermalized condition are not only affected by the vertical wind fluctuations but also by the density oscillations. Therefore, it may not be appropriate to infer air velocity oscillations from the ascent rate of these balloons.

In the present work we wish to extend and improve the one-dimensional results by the use of a model in a more general context, but which must be solved by numerical means due to the increased complexity. In fact, the search for a general analytical solution of an equation of motion like Eq. (1) is a difficult task (see, for example, the work by Coimbra and Rangel[26]). A code for the whole equation set was implemented in two dimensions,[11] the vertical and one

TABLE I. Input values for five runs.

Parameter	R	G	V	T	W
Gas thermodynamics	$T_g=T_a$	polytropic	$T_g=T_a$	$T_g=T_a$	$T_g=T_a$
Initial balloon and wind velocity	equal	equal	equal	equal	equal
Growth of velocity wave amplitude with height	saturated	saturated	saturated	saturated	saturated
History term in balloon equation	no	no	no	no	no
Maximum balloon volume (m^3)	12 000	12 000	12 000	12 000	12 000
Effective area of apex valve (m^2)	0.05	0.05	0.05	0.05	0.05
Initial balloon mass without gas and ballast (kg)	200	200	200	200	200
Initial ballast mass (kg)	100	100	100	100	100
Initial gas mass (kg)	52.5	49.5	52.5	52.5	52.5
Ballast flow during release (kg)	0.03	0.03	0.03	0.03	0.03
Gas molecular weight	4.00	4.00	4.00	4.00	4.00
Gas polytropic index	⋯	1.1	⋯	⋯	⋯
Gas initial temperature (K)	⋯	288	⋯	⋯	⋯
Initial balloon altitude (m)	0	0	0	0	0
Balloon flotation height (m)	22 000	22 000	22 000	22 000	22 000
Balloon flotation time (s)	3 000	3 000	3 000	3 000	3 000
Balloon minimum descent altitude (m)	0	0	0	0	0
Maximum w_b for ballast flow on ascent (m/s)	0	0	0	0	0
Minimum w_b for gas release on ascent (m/s)	10	10	10	10	10
Maximum w_b for ballast flow on descent (m/s)	−10	−10	−10	−10	−10
Minimum w_b for gas release on descent (m/s)	0	0	0	0	0
Mean u_a at sea level (m/s)	0	0	0	0	0
Mean u_a gradient in first atmospheric layer (l/s)	0	0	0.0008	0	0
Transition height of first and second layers (m)	10 000	10 000	10 000	10 000	10 000
Mean u_a shear in second atmospheric layer (l/s)	0	0	0.0008	0	0
Transition height of second and third layers (m)	20 000	20 000	20 000	20 000	20 000
Mean u_a shear in third atmospheric layer (l/s)	0	0	0.0008	0	0
Turbulent atmospheric dynamic viscosity (N s m^{-2})	0	0	0	0.0015	0
w'_a amplitude at the initial height (m/s)	0.1	0.1	0.1	0.1	0.6
Horizontal wavelength (m)	100 000	100 000	100 000	100 000	20 000
Vertical wavelength (m)	2 000	2 000	2 000	2 000	2 000
Wave phase constant (rad)	0	0	0	0	0
Initial time step for computation (s)	0.5	0.5	0.5	0.5	0.5
Minimum allowed stepsize (s)	0.1	0.1	0.1	0.1	0.1
Maximum allowed stepsize (s)	10	10	10	10	10
Required error tolerance	0.001	0.001	0.001	0.001	0.001
Storage intervals for ascent (s)	10	10	10	10	10
Storage intervals for flotation (s)	10	10	10	10	10
Storage intervals for descent results (s)	10	10	10	10	10
Maximum time allowed for computation (s)	12 000	12 000	12 000	12 000	12 000

relevant horizontal direction. The input specifies the flight, balloon, and atmosphere characteristics and the numerical requirements, whereas the output gives the simulation results in terms of time. The equation of motion depends on balloon geometrical and physical aspects and on the description of the atmosphere. There are in addition equations for the balloon thermodynamics and for the atmospheric thermal structure, flow field, and perturbations. We must then specify a variety of parameters to obtain solutions. The program run starts with four initial questions that correspond to some general matters that define the characteristics of the physical problem and its simulation, and they determine the corresponding subgroup from the whole equation set to be solved. The interrogations refer to: (1) gas behavior (thermalized or polytropic); (2) equal or different balloon and wind velocities at start (in the second case it adds some input parameters); (3) specification of gravity wave amplitude evolution with height (constant, saturated, or unsaturated increase); and (4) the inclusion or exclusion of an integral history term in the equation of motion. The last three questions will have the same answer throughout our simulations. For simplicity, a null differential velocity between balloon and atmosphere will be assumed at start to avoid the appearance of initial transient effects which are out of the scope of the present work. A saturated wave will be considered. The presence of the history term is the main inconvenience in the search for solutions of the equation of motion, but during typical balloon flights the Reynolds number is usually large, so it may be neglected[27] as we have done when writing Eq. (1). This leads to a substantial reduction of computer time.

If desired, the program user can easily alter the fixed reference values for the temperature, density, and molecular dynamic viscosity profiles in the air, so that other atmospheric conditions may be simulated, including the characteristics of other planets. The flight top altitude for the Earth cannot exceed 32 km due to the range of validity of the air temperature profile. Here, we will only modify the default reference values of the latter for some simulations on balloon oscillations, which depend on the corresponding gradient.

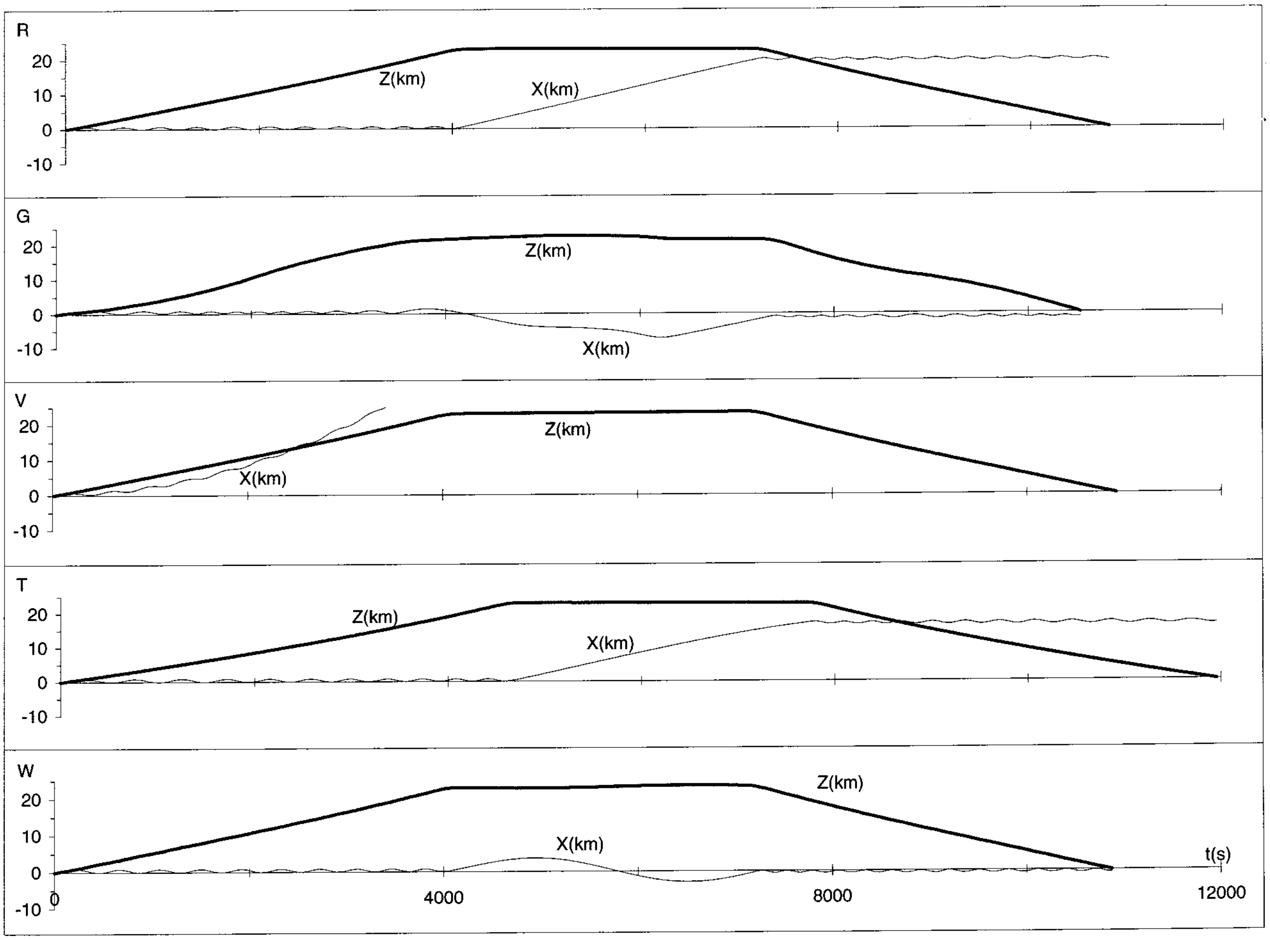

FIG. 1. Horizontal and vertical balloon position for runs R, G, V, T, and W.

III. NUMERICAL STUDY

A standard balloon flight essentially includes three stages: the ascent from the ground, flotation at some predetermined altitude, and the descent back to the surface. We are interested in a qualitative study of the basic characteristics of open balloon dynamics, which requires a set of solutions involving variation of parameters within the appropriate ranges. We will analyze the global dynamical behavior during flights, but we will mainly focus on the response to the local environmental stimuli. A full investigation would entail a very large number of runs. A series of simulations for a variety of physical situations has been designed here to provide general insight into balloon behavior in the terrestrial ambient. The scheme has been devised to provide maximum results with a minimum of computer runs. A typical reference case (R) is presented, where the gas thermalizes with the surrounding air and where the atmosphere includes a gravity wave but null background velocity and turbulent viscosity. It is possible to constrain within the model the alteration of primary balloon or atmosphere aspects to four basic independent changes: gas thermodynamics (G), ambient atmospheric velocity (V), air small-scale turbulence (T), and wave characteristics (W). The consideration of the most prominent variables of the problem involves therefore five cases, where the separate effects may be analyzed. In case G a polytropic gas evolution is assumed, where $\gamma_g = 1.1$. In case V a vertical shear for the background atmospheric horizontal velocity is employed. In case T an air turbulent viscosity is introduced. In case W the horizontal wavelength of the gravity wave was reduced (this leads to a shorter intrinsic period) and its amplitude was increased. The parameters of the input data file and their values for each run are shown in Table I. Molecular hydrogen is satisfactory for flights but helium has been considered here in all cases as it is often preferred due to certain advantages (e.g., nonflammable). Flotation time has been set in the five solutions to 3000 s at an altitude about 22 km (lower stratosphere). The chosen conditions ensure that no gas is lost by spillage during the whole flights.

In Fig. 1 it is possible to see the different trajectories followed by a balloon in the five cases. The fluctuating portions in the horizontal balloon coordinate X are induced by the wave. Oscillations in the vertical position Z are not so clearly discernible because they are less significant in terms of this variable (other dynamical parameters below will better highlight the balloon behavior in the vertical direction without a need to amplify the graph). During ascent or descent the mean vertical velocity dominates the displacement, so fluctuations can hardly be distinguished; something similar happens for X in case V, where fluctuations have a lower

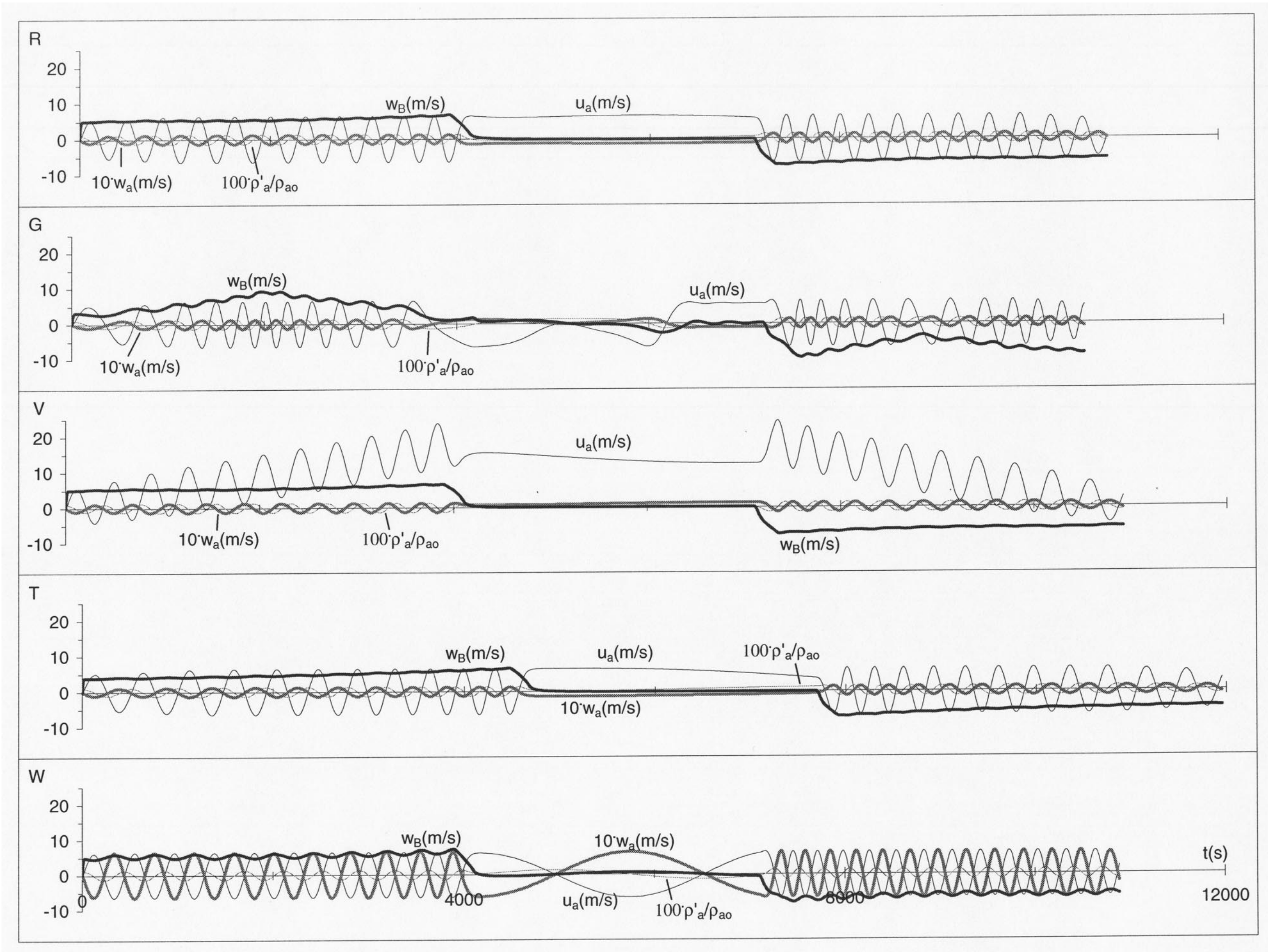

FIG. 2. Balloon vertical velocity and relevant atmospheric parameters at the current position for runs R, G, V, T, and W. Air density fluctuations and vertical velocity have been, respectively, multiplied by 100 and 10 for better visualization.

influence than the significant horizontal background wind. In addition, X grows out of graph bounds in V. At flotation, cases R, V, and T show a nearly flat balloon response; in G there is no clear pattern, whereas a part of a very weak oscillation may be seen in case W due to the stronger wave and because the intrinsic period has become short enough to be comparable with the time interval of this stage. The five solutions show some common characteristics mainly related to the global behavior in the vertical direction. The stay at negative horizontal coordinates during a significant part of the flight in two cases is circumstantial and has no special meaning, as this fact is only related to the preponderance of the wind encountered during the particular evolution. The analysis of additional dynamical variables becomes advisable for a better inspection of the problem. A further study of balloon dynamics will reveal some significant differences among the cases, as the balloon locally responds in each solution in a particular way.

In Fig. 2 we have represented for the five cases the balloon and air vertical velocities w_b and w_a ($\equiv w_a'$ fluctuations induced by the wave), the ratio of atmospheric density oscillations and background value ρ_a'/ρ_{ao}, and the air horizontal velocity u_a ($\equiv u_a'$ except in V). The latter may be taken as representative of the balloon horizontal velocity (this one exhibits nearly the same behavior but it possesses slightly smaller peaks and troughs with a mean delay of 10 s, which implies an average shift of roughly 10° in the oscillations, as the average wave period as seen from the balloon is about 350 s, with small differences among the five cases). The atmospheric values correspond to the current balloon position. Some variables have been multiplied by factors for better visualization of all of them in the same graph. The relative phase lags of ρ_a'/ρ_{ao} and u_a' with respect to w_a' are 90° and 180° (the wave field phase at the balloon position is $\phi = \omega t - k_x X - k_z Z + C$, where ω, k_x, k_z are the angular frequency and horizontal and vertical wavelengths, whereas C is a different constant for each variable), which follows from the polarization relations for gravity waves by Hines[17] in the appropriate limits.[11] The balloon exhibits in all cases a small horizontal displacement with respect to the background wind, which is the intrinsic wave system, during ascent or descent (see Fig. 1). In addition, its typical vertical velocity is much larger than the wave vertical phase speed (0.65 m/s for W and 0.13 m/s in the remaining cases), so a vertical cut from an almost frozen-in wave is seen during those stages.

In all cases but G the balloon vertical velocity may be approximately described throughout most of the ascent and descent by a constant plus a superposition of fluctuations and

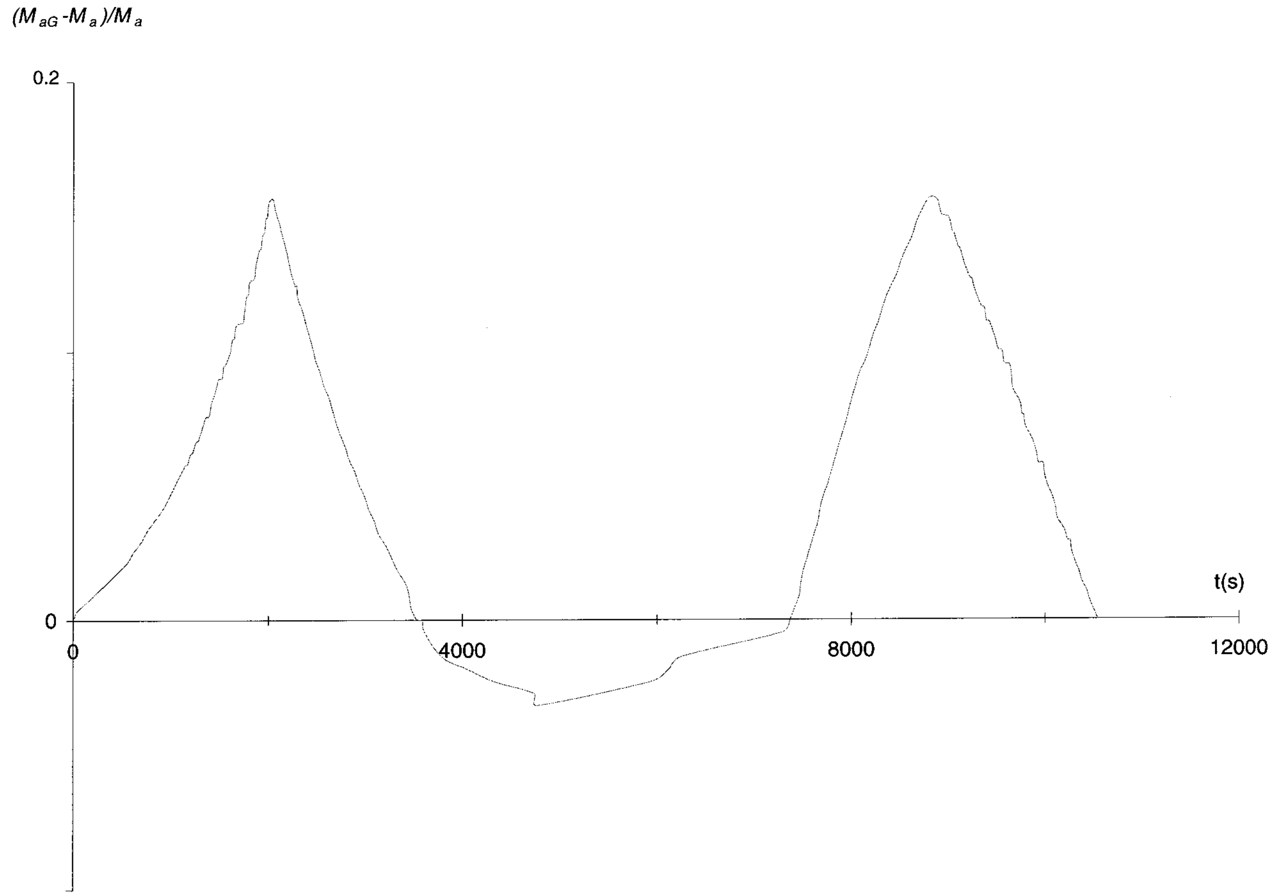

FIG. 3. Fractional thermal drag or push on a balloon for a polytropic process (G) with respect to the case where the gas thermalizes with the surrounding air.

at flotation by an almost null value. Even though diverse variables of the problem change significantly, the balloon rises (falls) during ascent (descent) at a nearly uniform rate in those four solutions. The dominant vertical forces are buoyancy (weight and upthrust) and drag, which nearly cancel each other (see below). Setting one equal to the other, assuming in agreement with simulations and observations that $|\underline{w_b}| \gg |w_a|$ and $u_b \approx u_a$ and decomposing w_b into a constant $\overline{w_b}$ plus a small perturbation gives

$$\pm \tfrac{1}{2} C_D A \rho_a \overline{w_b^2} = (M_a - M_b) g, \tag{6}$$

where $\pm$ refers to ascent and descent. For a spherical balloon and taking into account Eq. (2), one obtains

$$\overline{w_b} = \pm \sqrt{\frac{8g(3V/4\pi)^{1/3}}{3C_D}\left|1 - \frac{M_b}{M_a}\right|}. \tag{7}$$

Although $|w_b| \gg |w_a|$ may not be strictly valid at flotation, notice that $M_b = M_a$ gives $\overline{w_b} = 0$. From $V = M_g/\rho_g$, replacing ρ_g in terms of pressure and temperature through Eq. (4), recalling that gas and air pressure are in equilibrium at the bottom balloon opening and that $T_g = T_a$ in the thermalized cases, which also implies[11]

$$M_a = M_g \frac{W_a}{W_g}, \tag{8}$$

also considering that for these balloon flights $\mathrm{Re} > 4.5 \times 10^5$ during ascent or descent, i.e., $C_D = 0.3$, and replacing p/T_a through Eq. (4) for air instead of the balloon gas, we get from Eq. (7)

$$\overline{w_b} = \pm \sqrt{\frac{8g(3M_g W_a/4\pi W_g \rho_a)^{1/3}}{0.9}\left|1 - \frac{M_b W_g}{M_g W_a}\right|}. \tag{9}$$

Only ρ_a may vary on the right-hand side if no gas is lost and it could do so by slightly more than one order of magnitude in the altitude range considered here. However, it will become moderated because it is raised to the power of 1/6 and therefore $\overline{w_b}$ will be nearly a constant.

In order to explain the different shape exhibited by the vertical balloon velocity profile in case G, we need to replace now the mass of displaced air by the expression appropriate for a polytropic process[11]

$$M_{a_G} = M_g \frac{W_a T_g}{W_g T_a} \tag{10}$$

in Eq. (7), so

$$\overline{w_b} = \pm \sqrt{\frac{8g(3M_g W_a/4\pi W_g \rho_a)^{1/3}}{0.9}\left|1 - \frac{M_b W_g T_a}{M_g W_a T_g}\right|}, \tag{11}$$

which shows an additional dependence on T_g/T_a as compared to the thermalized condition. Notice again that $\overline{w_b}$

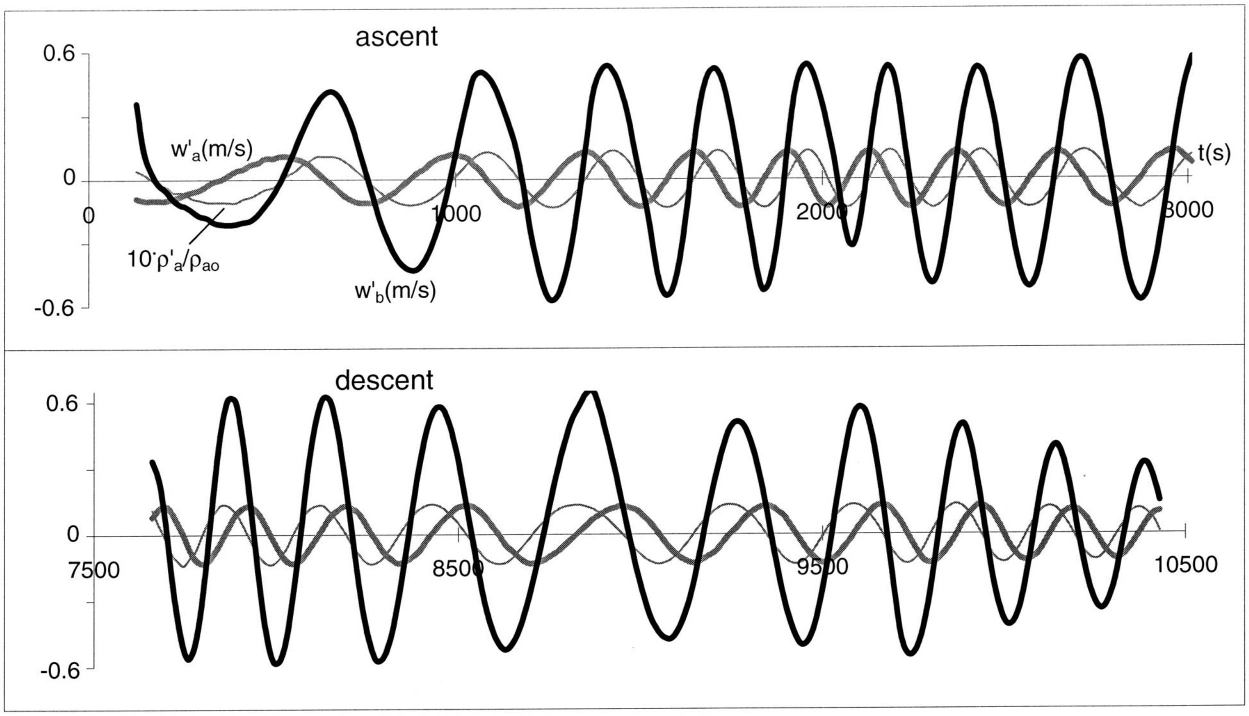

FIG. 4. Fluctuations of balloon vertical velocity and air density and vertical velocity during ascent and descent in a polytropic process (G). Air density fluctuations have been multiplied by 10 for better visualization.

$=0$ at flotation. In order to work out the influence of the temperature ratio we compare both expressions introduced for M_a [Eqs. (8) and (10)]

$$\frac{M_{a_G}-M_a}{M_a}=\frac{T_g}{T_a}-1=\frac{W_g}{W_a}\,\frac{\rho_a}{(p/c)^{1/\gamma_g}}-1,$$

where we have used Eqs. (4) and (5) and the condition of equal air and gas pressures in the second equality. The fractional thermal drag or push of the polytropic process with respect to the thermalized condition depends on the relative evolution of the air and gas temperatures, and it may be also expressed exclusively in terms of air variables. We represent it in Fig. 3 (fluctuations were excluded for the calculations). We may now explain the different global behavior of the vertical balloon velocity in Fig. 2(b), as it is straightforward to relate the global extremes in both figures [notice that the second term in the absolute value in Eq. (11) is smaller and larger than 1, respectively, during ascent and descent].

Regarding the w_b fluctuations, in the four cases where $T_g=T_a$ the balloon emulates the wind very closely (there are also slightly smaller peaks and troughs here and a mean delay of 10 s, i.e., an average shift of roughly 10° in the oscillations). Only in the polytropic process does a significant difference in phase and amplitude exist; see Fig. 4. Moreover, balloon oscillations become larger than the wind ones due to the effect of air density fluctuations (up to four times). It may be seen that w_b maxima are contained in the 90° interval between ρ_a and w_a peaks, but on average 60 s away from the latter ones, which approximately represents a 70° separation in the corresponding oscillations. Almost contrary to the thermalized condition, the balloon vertical velocity fluctuations now follow the air density rather than the vertical velocity. Therefore, it is only apparent that the balloon motion leads the air in the vertical direction in the last stage of the flight (density peaks occur 90° ahead of air vertical velocity maxima due to increasing ϕ during descent in the present simulation). This was all qualitatively predicted by our simple analytical 1D model;[25] the present solutions lead to more accurate and complete results.

An explanation for the different response to air fluctuations in both thermal cases requires the analysis of the basic dynamics of the problem, i.e., the force or acceleration terms. From Fig. 2 it is clear that the most interesting behavior occurs during ascent or descent, so we will restrict ourselves to these stages. Horizontal and vertical drag are very rich terms because they couple thermodynamics and horizontal and vertical dynamics of air and balloon [see Eq. (1)]. In Fig. 2 we see that the balloon velocity period is the same in the horizontal and vertical direction, otherwise new frequencies could have been generated by the nonlinear interaction. There have been previous studies of the effects of the nonlinear term by standard expansion methods, but they were applied to cases with particular time or spatial air variations, which permitted some simplifications by symmetry arguments.[6,28]

In Fig. 5 we may see the relevant horizontal and vertical acceleration terms. The former direction is clearly dominated in all cases by the corresponding drag component. Only in case W does the dynamic term have some relevance due to the stronger wave. In addition, it is quite peculiar that the phase difference between the horizontal drag and dynamical term exhibits in all cases a 180° shift between ascent and

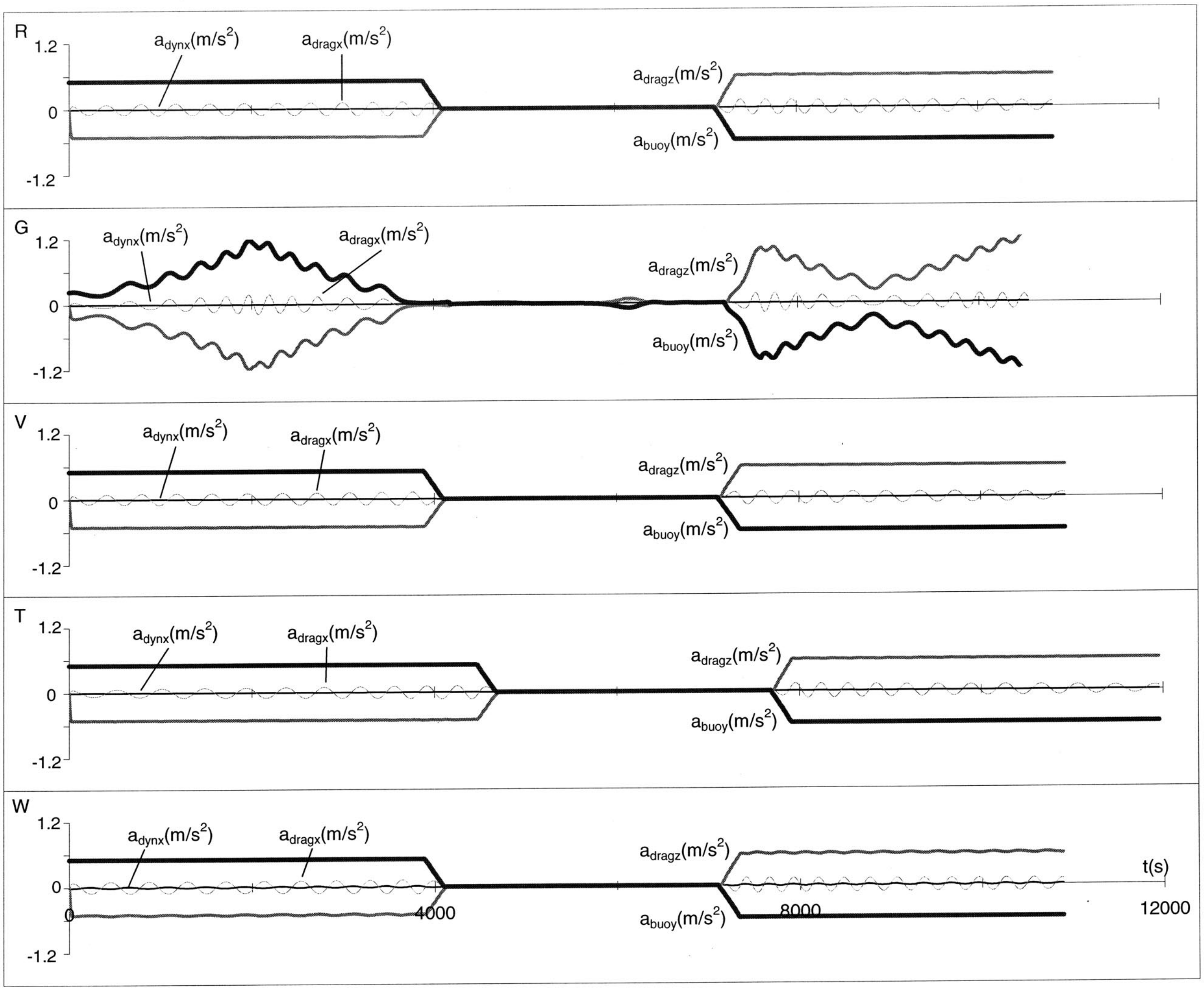

FIG. 5. Relevant terms of the horizontal and vertical balloon acceleration for runs R, G, V, T, and W.

descent (clearly seen in W). The former is governed as stated below by the horizontal differential velocity between balloon and air, but the latter depends on how the balloon goes through the phase of the wave field ϕ, which shows nearly opposite evolution in both stages; see, for example, case R in Fig. 6. The vertical acceleration in the thermalized cases during ascent and descent is originated by a constant buoyancy minus the drag, which may be visualized as the same constant plus an oscillation (the fluctuation becomes more important in W). Vertical drag oscillations are relevant in case G and buoyancy variations slightly precede them. Also, due to the wave strength only in case W does the dynamical acceleration exhibit some preponderance during flotation and, as it is proportional to $D\mathbf{v}_a/dt$, it produces a forcing which causes the balloon to follow not only the horizontal but also the vertical fluid velocity (see Figs. 1, 2). It is also easier to see this due to the shorter wave period in case W.

The dominating vertical components of the equation of motion are buoyancy and drag. The balloon vertical velocity at zeroth order was given in Eqs. (9) and (11). At first order (primed variables) from Eq. (1) we have

$$(M_b+\tfrac{1}{2}M_a)\dot{w}_b'=gM_a'-(\tfrac{1}{2}C_DA\rho_a|\mathbf{v}_b-\mathbf{v}_a|(w_b-w_a))', \tag{12}$$

where

$$M_a'=\rho_a'V+\rho_aV', \tag{13}$$

and $\dot{w}_b$ refers to the time derivative of w_b.

In the thermalized cases the mean vertical drag acceleration nearly counterbalances the constant buoyancy component (see Fig. 5). The mass of displaced air is not affected by atmospheric variations and does not change [unless M_g becomes altered during the flight; see Eq. (8)], but the drag does not remain constant. For Re$>4.5\times10^5$ C_D stays the same and we notice by Eqs. (8) and (13) that $V'/V=-\rho_a'/\rho_a$, i.e., A'/A and ρ_a'/ρ_a will nearly cancel each other, so Eq. (12) yields

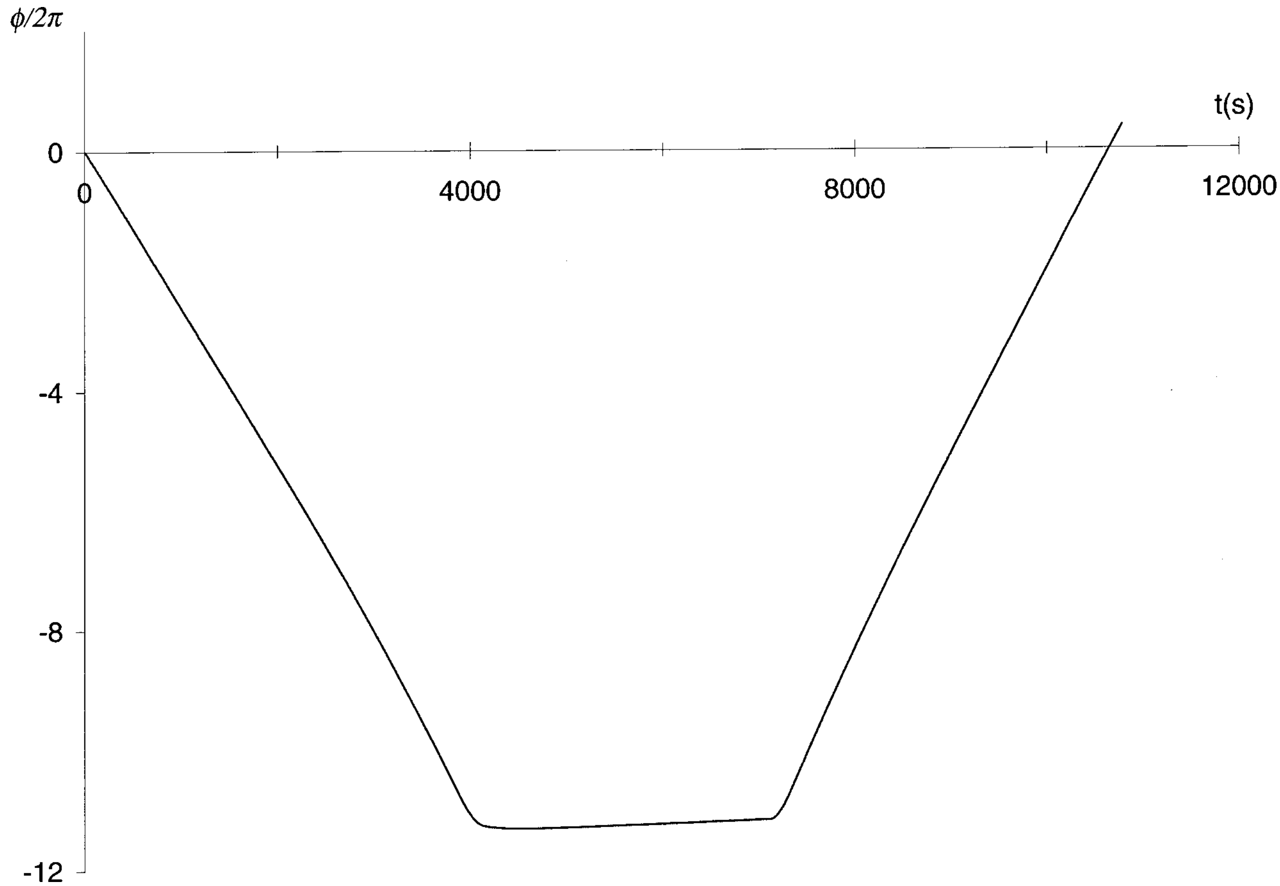

FIG. 6. Wave-field phase at the balloon position for the reference case (R).

$$\left(M_b+\frac{1}{2}M_a\right)\dot{w}_b'$$

$$\approx -\frac{1}{2}C_D A\rho_a(|w_b-w_a|(w_b-w_a))'$$

$$=\mp\frac{1}{2}C_D A\rho_a((w_b-w_a)^2)'\approx\mp C_D A\rho_a\overline{w}_b(w_b'-w_a')$$

$$=-\frac{2(M_a-M_b)g}{\overline{w}_b}(w_b'-w_a'),$$

where $\mp$ refers to ascent and descent and we considered that $u_b \approx u_a$, $|w_b| \gg |w_a|$ and $\overline{w}_b >$ and <0, respectively, for these stages and we used Eq. (6) for the last equality. As M_a does not change during ascent or descent if M_g stays the same and $\overline{w}_b$ does not vary significantly, it may be seen that for a positive constant k

$$\dot{w}_b' = -k(w_b'-w_a'),$$

so w_b' will be determined by a kind of perturbation drag, which will compel it to follow w_a' (replacing typical values, k defines a characteristic time of 0.01 s, which is much shorter than the wave period seen from the balloon). The small departures between w_b' and w_a' in the numerical results may be attributed to the influence on drag of coupling with the horizontal direction through the square root term, minor vertical acceleration terms like the dynamical component, or inertia. Similarly, the small difference between u_b' and u_a' may correspond to coupling with the vertical direction or inertia, but this statement also applies for the polytropic case.

The vertical drag oscillations nearly counterbalance the buoyancy ones for the polytropic evolution (see Fig. 5). Changes in $M_b+1/2M_a$ (M_a is variable now) are negligible. Buoyancy fluctuations depend on air density and gas volume oscillations [see Eq. (13)]. For a polytropic gas the fluctuations induced by a gravity wave lead to $V'=0$,[11] so

$$M_a' = \rho_a' V.$$

However, even a small density oscillation will produce a relatively large buoyancy variation because it is given by the difference between upthrust and weight, which is much smaller than the former itself. From Eq. (12) we may infer that an interplay between air vertical velocity and density influences on balloon vertical velocity fluctuations, respectively, through the drag and upthrust perturbation terms exists. Moreover, $A'=0$ as $V'=0$ and following steps similar to those above

$$\left(M_b+\frac{1}{2}M_a\right)\dot{w}_b'$$

$$=g\rho_a' V-\frac{1}{2}C_D A(\rho_a|w_b-w_a|(w_b-w_a))'$$

$$\approx g\rho_a' V\left(1-\frac{M_a-M_b}{M_a}\right)-\frac{2(M_a-M_b)g}{\overline{w}_b}(w_b'-w_a')$$

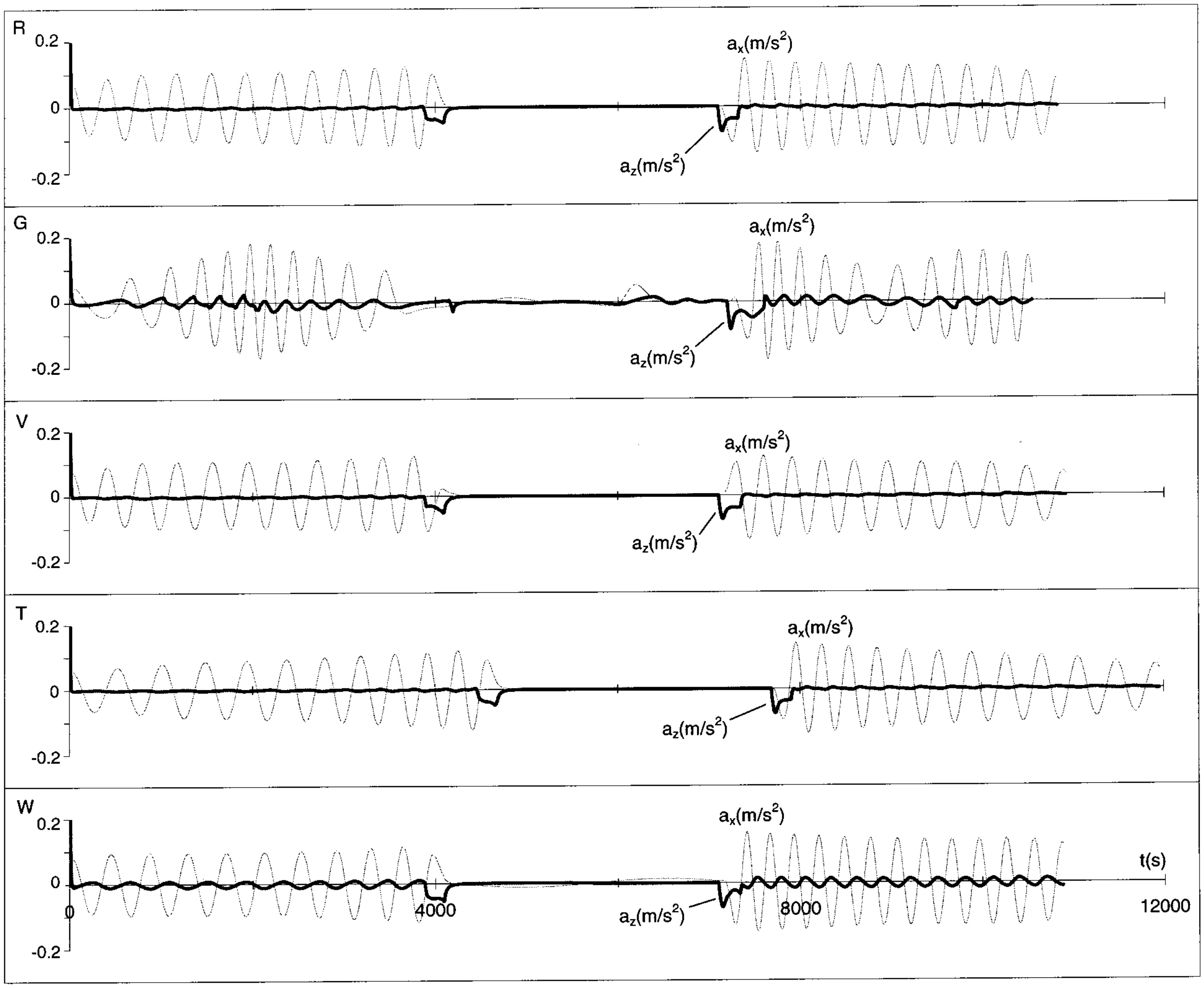

FIG. 7. Horizontal and vertical balloon acceleration for runs R, G, V, T, and W.

$$\approx g\rho_a' V - \frac{2(M_a - M_b)g}{\overline{w}_b}(w_b' - w_a'),$$

as usually $|(M_a - M_b)/M_a \ll 1|$. Replacing typical values shows that the first term on the right-hand side is larger than the second; therefore, w_b' peaks are greater and closer to ρ_a' ones than in the thermalized case.

The total vertical and horizontal accelerations may be seen in Fig. 7 and are, respectively, given essentially by the difference between vertical drag and buoyancy and by the horizontal drag component. In the thermalized cases the drivers of each component then are $w_b' - w_a'$ (explained above) and $u_b - u_a$ (it is the only factor of the horizontal drag which may alternate positive and negative values). The first and second differential velocities will, respectively, be in phase quadrature with w_a and u_a (the difference between two slightly shifted oscillations will have the same shape but different amplitude 90° out of phase with them), which are 180° apart in a gravity wave. The same separation applies then to both drivers, and the total horizontal and vertical accelerations will therefore have a 180° phase difference. In the polytropic case, while the total horizontal component shows a similar behavior, the vertical component is given by the difference of two similar sinusoidal curves slightly shifted, buoyancy and drag, but now w_b' follows ρ_a' rather than w_a', which in a gravity wave are in phase quadrature. The average shift between the balloon horizontal and vertical accelerations would be expected around 90° and calculations found 97°.

We may set up simple analytical solutions to describe the ascent or descent of an open balloon when its gas thermalizes with the surrounding air (the same is not as easy for the polytropic case)

$$u_b = u_a,$$

$$\overline{w}_b = \pm\sqrt{\frac{8g(3M_g W_a/4\pi W_g \rho_a)^{1/3}}{0.9}\left|1 - \frac{M_b W_g}{M_g W_a}\right|},$$

$$w_b' = w_a',$$

where the air variables are all known functions of space and time.

Analytical solutions have already been found for flotation under simple conditions. The ratio of quadratic and linear drag is [see Eqs. (1) and (3)]

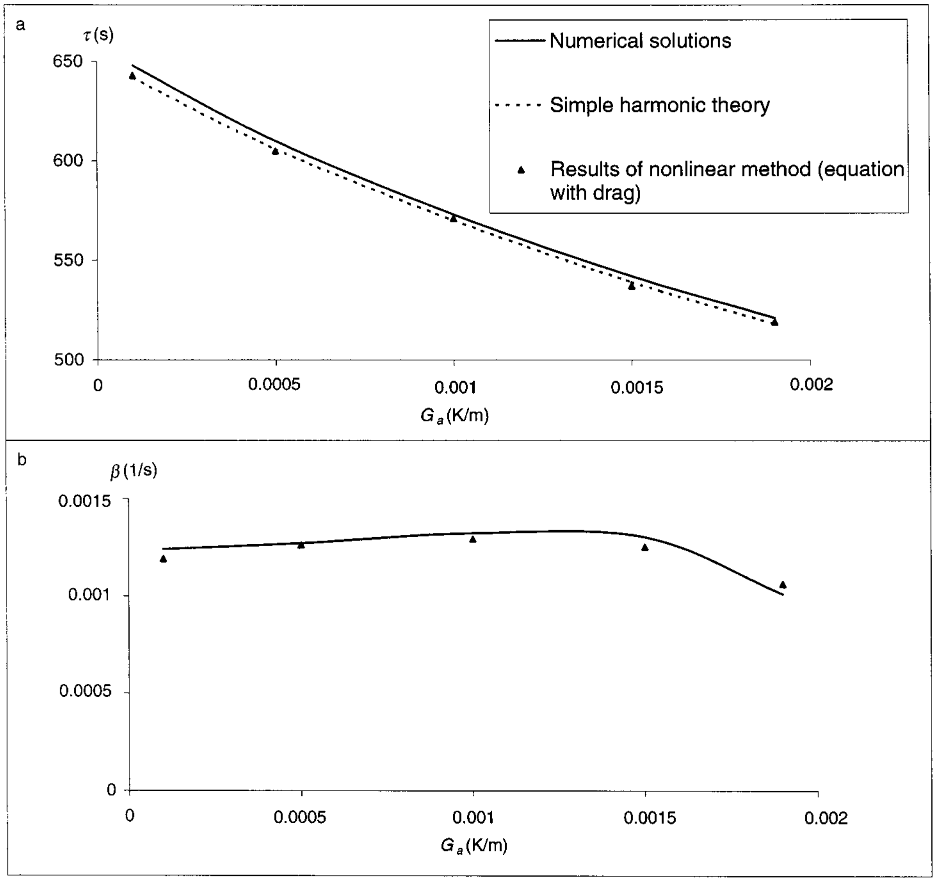

FIG. 8. (a) Balloon oscillation period in a polytropic process after reaching flotation vs air temperature gradient at this altitude. (b) Damping factor of the decaying oscillations.

$$\frac{\frac{3}{16}\frac{|\mathbf{v}_b-\mathbf{v}_a|^2}{r}}{\frac{9}{2}\frac{\nu|\mathbf{v}_b-\mathbf{v}_a|}{r^2}}=\frac{\mathrm{Re}}{48},$$

where $\nu=\mu/\rho_a$ is the air kinematic viscosity, Re is of the order of 10^4 ($v\sim 1$ m/s, $r\sim 10$ m, $\nu\sim 10^{-5}$ m^2/s). The buoyancy term is proportional to the displacement from the equilibrium height. The equation near flotation in a stationary atmosphere may be written as[11]

$$\ddot{Z}+\alpha\dot{Z}|\dot{Z}|+\Omega^2(Z-Z_e)=0,$$

with

$$\alpha=\frac{1}{8r},$$

$$\Omega^2=\frac{2}{3}\frac{g(G_a-G_g)}{T_{a_{m_e}}},$$

where G means vertical temperature gradient [G_a is given by the program and $G_g=g(1-1/\gamma_g)/R_a$ for the polytropic case] and $T_{a_{m_e}}$ is the air mean temperature at the equilibrium altitude Z_e. The balloon approaches an asymptotic altitude for thermalized conditions ($\Omega=0$) or it oscillates for polytropic behavior. The fluctuations become damped and the period τ exhibits some small changes with time. Approximate solutions have been found for this type of nonlinear damping by an asymptotic method,[29] which are valid for negligible fluid velocity and where the smallness parameter is given by $4\alpha A_o$ (A_o is the initial oscillation amplitude), which in this case is A_o/d, d being the balloon diameter. We have numerically studied the variation of balloon oscillation after reaching flotation altitude with air temperature gradient in the absence of wind and waves, and compared it to the results obtained by simple undamped harmonic motion theory [see Fig. 8(a)]. Although the small differences between the period calculated from theory and the average of the numerical results may be attributed to the nonlinear drag, the asymptotic method produces corrections that are much smaller than the discrepancies, or may even introduce changes in the opposite direction. However, differences fall within the variability of the period of the decaying oscillations. The failure can probably be attributed to the fact that the smallness parameter ranges around 1. Nevertheless, calculations of the damping factor $1/(1+\beta t)$ are in agreement with the fits to the decays in the numerical results [see Fig. 8(b)]. The oscillations described here may be seen only in the last part of flotation of case G [Fig. 2(b)], because the presence of wind and waves forces a transition after the end of the ascent. Analytical solutions for a nonstationary atmosphere have apparently not been obtained. If it was possible to obtain them in the presence of a gravity wave we know that the balloon should approximately follow the vertical wind oscillations due to the drag, except in case G due to buoyancy effects (see Fig. 2).

The present runs are in accordance with the characteristics of four open stratospheric balloons launched in Mendoza

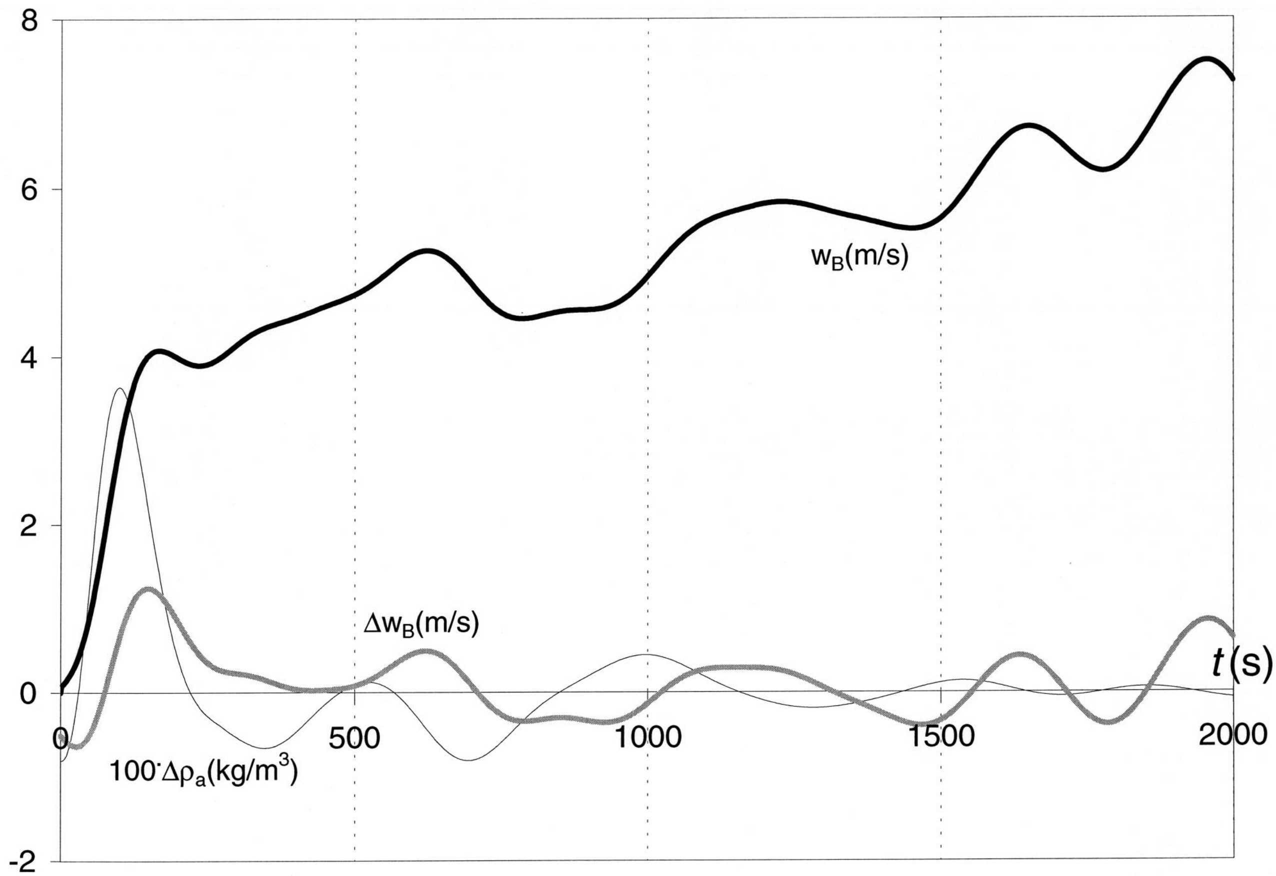

FIG. 9. Balloon vertical velocity, its fluctuating component, and atmospheric density data from an open atmospheric balloon sounding.

(32°S, 68°W) near the Andes Mountains in 1990 by the Ports sounding campaign,[4] which was supported by the French Balloon Program of the Center National d'Etudes Spatiales. The total mass of the load, skin, and ballast ranged between 300 and 500 kg and the gas mass between 45 and 55 kg in the different flights. In order to evaluate the ability of our simulations to reproduce some features, we show and outline some experimental observations from the Ports campaign. Data from the second flight, which exhibit the highest quality among the four, have been processed and are presented in Fig. 9. A dominant nearly steady gravity wave was clearly detected during this sounding. A large part of the ascent is shown. The descent shows similar features, whereas the flotation exhibits as little variation in the variables as would be expected from the thermalized cases in Fig. 2. Atmospheric density, balloon vertical velocity, and its fluctuating component are shown in the graph. The air vertical velocity could not be included because it cannot be reconstructed with the required accuracy due to uncertainties in the balloon tracking system and because of motions induced in the gondola. Perturbations in the graph are not exactly sinusoidal but types of oscillatory patterns can be easily observed. It may be clearly seen that the balloon responds almost exactly 90° out of phase to the density. This behavior coincides with the expectations on the air vertical velocity (both quantities are in phase quadrature in a gravity wave). The balloon and wave velocity agreement corresponds to the thermalized case. This has a close relation to some characterisitcs of the skin, where the relevant values for the flight are: thermal conductivity $=0.40$ W/(K m) and thickness$=25$ μm. Applying these data to a recent study[30] where a relationship between skin thermal conductivity and polytrope for open balloons has been obtained gives a polytropic index about 1, which in fact implies an infinite heat transfer.

IV. CONCLUSIONS

Examination of the numerical results from five cases contemplating gas thermodynamics, air background velocity, atmospheric small-scale turbulence, and gravity wave characteristics revealed a number of significant features of open atmospheric balloon behavior. The horizontal motion exhibited some small but observable deviations from the corresponding wind component. The vertical response becomes determined by the vertical wind fluctuations when the gas thermalizes with the surrounding air, and under a polytropic gas evolution the air density fluctuations become important. The run with a stronger wave exhibits some distinctive characteristics, but only the polytropic solution emerges among the five examples as a significantly different case regarding the general behavior of balloon velocity and acceleration. We therefore conclude that these dynamical aspects are not only governed by the atmospheric environment but also by the no less important factors determining the gas thermal behavior. In the presence of gravity waves it would be possible to infer the thermal characteristics from the phase difference between vertical and horizontal balloon velocity or acceleration or between air and balloon vertical velocity. Simple approximate analytical solutions for balloon motion may be found, except for the ascent or descent with a polytropic gas. Further simulations showed that the natural oscillatory frequency near flotation of an open balloon with a polytropic gas is very close to the calculation by simple harmonic motion. The damping nearly coincides with the predictions of an asymptotic method for nonlinear equations. We finally compared the simulation results with observations from a sounding and noticed that the balloon exhibited a behavior close to the one expected for infinite heat transfer between gas and surroundings, which is in agreement with the characteristics of the used balloon.

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[1]P. Alexander and A. de la Torre, ''The interpretation of saturated spectra as obtained from atmospheric balloon measurements,'' J. Appl. Meteorol. **38**, 334 (1999).
[2]P. Morel and W. Bandeen, ''The EOLE experiment: Early results and current objectives,'' Bull. Am. Meteorol. Soc. **54**, 298 (1973).
[3]M. Kitchen and G. J. Shutts, ''Radiosonde observations of large-amplitude gravity waves in the lower and middle stratosphere,'' J. Geophys. Res. **95**, 20451 (1990).
[4]A. de la Torre, H. Teitelbaum, and F. Vial, ''Stratospheric and tropospheric wave measurements near the Andes mountains,'' J. Atmos. Terr. Phys. **58**, 521 (1996).
[5]J. H. Hirsch and D. R. Booker, ''Response of superpressure balloons to vertical air motions,'' J. Appl. Meteorol. **5**, 226 (1965).
[6]G. D. Nastrom, ''The response of superpressure balloons to gravity waves,'' J. Appl. Meteorol. **19**, 1013 (1980).
[7]B. Trounday, L. Perthuis, S. Strebelle, J. D. Ferrara, and C. R. Mechoso, ''Dispersion properties of the flow in the southern stratosphere during winter and spring,'' J. Geophys. Res. **100**, 13901 (1995).
[8]F. Kreith and J. F. Kreider, *Numerical Prediction of the Performance of High Altitude Balloons* (NCAR, Boulder, 1974).
[9]J. M. Cameron, I. S. Smith, J. A. Cutts, S. M. Raqué, J. A. Jones, and J. J. Wu, ''Versatile modeling and simulation of Earth and planetary balloon systems,'' AIAA International Balloon Technology Conference, AIAA 99-3876 (1999).
[10]G. D. Nastrom, ''Reply,'' J. Appl. Meteorol. **20**, 1091 (1981).
[11]P. Alexander and A. de la Torre, ''A program for the simulation and analysis of open atmospheric balloon soundings,'' Comput. Phys. Commun. **151**, 96 (2003).
[12]L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, London, 1959).
[13]V. L. Streeter, *Handbook of Fluid Dynamics* (McGraw-Hill, New York, 1961).
[14]G. K. Batchelor, *An Introduction to Fluid Dynamics* (Cambridge University Press, London, 1970).
[15]M. R. Maxey and J. J. Riley, ''Equation of motion for a small rigid sphere in a nonuniform flow,'' Phys. Fluids **26**, 883 (1983).
[16]*U.S. Standard Atmosphere, 1976* (U.S. Government Printing Office, Washington, D.C., 1976).
[17]C. O. Hines, ''Internal atmospheric gravity waves at ionospheric heights,'' Can. J. Phys. **38**, 1441 (1960).
[18]J. Barat and C. Cot, ''Wind shear rotary spectra in the atmosphere,'' Geophys. Res. Lett. **19**, 103 (1992).
[19]A. de la Torre, P. Alexander, and A. Giraldez, ''Large amplitude waves detected with balloons near the Andes mountains,'' Geophys. Res. Lett. **23**, 1103 (1996).
[20]J. Pedlosky, *Geophysical Fluid Dynamics* (Springer, New York, 1987).
[21]A. L. Morris, *Scientific Ballooning Handbook* (NCAR, Boulder, 1975).
[22]S. M. Raqué, SINBAD 3.0, *Nasa's Scientific Balloon Analysis Model, User's Manual* (NASA, Wallops Island, 1993).
[23]S. M. Raqué, ''A spreadsheet tool for terrestrial and planetary balloon design,'' AIAA International Balloon Technology Conference, AIAA 99-3878 (1999).
[24]W. J. Anderson and I. Taback, ''Oscillation of high-altitude balloons,'' J. Aircr. **28**, 606 (1991).
[25]P. Alexander, J. Cornejo, and A. de la Torre, ''The response of an open stratospheric balloon in the presence of inertio-gravity waves,'' J. Appl. Meteorol. **35**, 60 (1996).
[26]C. F. M. Coimbra and R. H. Rangel, ''General solution of the particle momentum equation in unsteady Stokes flow,'' J. Fluid Mech. **370**, 53 (1998).
[27]F. B. Tatom and R. L. King, *Determination of Constant-volume Balloon Capabilities for Aeronautical Research* (Science Applications, Huntsville, 1976).
[28]G. D. Nastrom and T. E. VanZandt, ''An analytical study of nonlinear responses of rising balloons to horizontal winds,'' J. Appl. Meteorol. **21**, 413 (1981).
[29]N. Minorsky, *Nonlinear Oscillations* (Van Nostrand, Princeton, 1962).
[30]A. de la Torre, P. Alexander, and J. Cornejo, ''A relationship between skin thermal conductivity and gas polytropic index in an open atmospheric balloon,'' J. Appl. Meteorol. **42**, 325 (2003).