

Primordial black hole evolution in two-fluid cosmology

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ABSTRACT

Several processes in the early Universe might lead to the formation of primordial black holes with different masses. These black holes would interact with the cosmic plasma through accretion and emission processes. Such interactions might have affected the dynamics of the Universe and generated a considerable amount of entropy. In this paper, we investigate the effects of the presence of primordial black holes on the evolution of the early Universe. We adopt a two-fluid cosmological model with radiation and a primordial black hole gas. The latter is modelled with different initial mass functions taking into account the available constraints over the initial primordial black hole abundances. We find that certain populations with narrow initial mass functions are capable to produce significant changes in the scalefactor and the entropy.

Key words: black hole physics – early Universe – cosmology: miscellaneous.

1 INTRODUCTION

There is overwhelming evidence supporting the existence of black holes in the Universe. Studies of stellar and gas dynamics strongly suggest the presence of supermassive black holes ($M_{\text{BH}} \sim 10^6\text{--}10^9 M_{\odot}$) at the centre of most galaxies (e.g. Ferrarese & Ford 2005). At smaller scales, stellar-mass black holes ($M_{\text{BH}} \sim 3\text{--}10 M_{\odot}$) are thought to be the result of the collapse of massive stars (Neugebauer 2003). Their presence is manifested in X-ray binaries (XRBS). There are currently about 60 stellar black hole candidates (Corral-Santana et al. 2016). Additionally, recent gravitational wave detections have revealed the existence of binary systems of black holes with several tens of solar masses at moderate redshifts (Abbott et al. 2016a,b, 2017).

Under the extreme conditions of the early Universe, black holes can be formed from direct collapse (Zel'dovich & Novikov 1966; Hawking 1971; Carr & Hawking 1974). These are called *Primordial black holes* (PBHs) and have been extensively investigated (see Khlopov 2010 for a review).

A PBH would form with a mass of order the horizon mass $M_{\text{H}}(t)$,

$$M_{\text{PBH}} \sim M_{\text{H}}(t) \sim \frac{c^3 t}{G} \sim 10^{15} \left(\frac{t}{10^{-23}\text{s}} \right) \text{g}, \quad (1)$$

as can be seen by a simple comparison between the cosmic density at time t after the big bang and the density associated with a black hole of mass M . This implies that PBHs could span a wide range

of masses. In particular, they might be small enough for Hawking radiation to be important (Hawking 1974).

PBH formation requires the existence of large inhomogeneities in the early Universe (Carr & Hawking 1974; Carr 1975). Independently of the source of these inhomogeneities, the formation can be enhanced by some processes, such as phase transitions – for example, from bubble collisions (Crawford & Schramm 1982; Hawking, Moss & Stewart 1982), collapse of cosmic strings (Hawking 1987; Polnarev & Zembowicz 1991) or domain walls (e.g. Berezin, Kuzmin & Tkachev 1983; Caldwell, Chamblin & Gibbons 1996) – or a sudden reduction in the pressure at the quark-hadron era (Jedamzik 1997; Jedamzik & Niemeyer 1999). Furthermore, applications of ‘critical phenomena’ to PBH formation suggest that their spectrum could go well below the horizon mass (e.g. Niemeyer & Jedamzik 1998; Green & Liddle 1999). PBH formation can also occur in a matter-dominated universe (see, e.g. Khlopov & Polnarev 1980; Polnarev & Khlopov 1985). For more details on all these mechanisms the reader is referred to Carr et al. (2010).

Although PBHs have not been detected so far, their study covers several areas of interest: baryogenesis (Barrow 1980; Lindley 1981; Barrow, Copeland & Kolb 1991a; Hook 2014), dark matter (see Carr, Kühnel & Sandstad 2016b, and references therein), big bang nucleosynthesis (Zeldovich et al. 1977; Vainer, Dryzhakova & Naselskii 1978; Kohri & Yokoyama 2000), reionization of the Universe (Gibilisco 1998) and gravitational waves (Bird et al. 2016; Sasaki et al. 2016).

Several constraints have been imposed on the initial number of PBHs formed (see Carr et al. 2010 for a review). Most of these limits

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are related to the different potential interactions between PBHs and other astrophysical objects (e.g. gravitational interactions) and the observables resulting from black hole evaporation. The importance of these constraints is that they indirectly impose restrictions on the conditions of the early Universe and, hence, on different early universe models (e.g. inflation models, Josan & Green 2010; Peiris & Easter 2008). However, not all constraints on PBHs are equally reliable. For example, at the lowest masses ($M \lesssim 10^6 \text{g}$) the only available constraint relies on a strong assumption, namely that black holes do not completely evaporate but leave behind Planck-mass relic particles (MacGibbon 1987).

The presence of a PBH population in the early Universe could have affected the cosmic evolution directly. The main feature of such a population is its initial mass function (IMF). According to the particular mechanism and time-scale of the formation process, this IMF can either extend over a wide mass range or be narrow and centred on a certain mass. Barrow et al. (1991b) studied the cosmic evolution of the early Universe considering radiation and a population of PBHs with a power-law IMF; they assume that the two components interacted only through Hawking evaporation. Other scenarios considered in later works involve PBH populations with narrow or *monoenergetic* IMFs (e.g. Barrow, Copeland & Liddle 1992; Zimdahl & Pavón 1998; Brevik & Halmes 2003) or additional space–time dimensions (Borunda & Masip 2010).

In this work, we investigate the early Universe evolution considering PBH populations with both extended and narrow IMFs, taking into account the best available constraints on PBH abundances in the characterization of the scenarios. We consider an FLRW space–time and a two-perfect-fluid model: a PBH gas with a dust-like equation of state and a relativistic component (radiation). The fluids exchange energy through Hawking evaporation and accretion on to the black holes. The energy exchange is coupled to the metric scalefactor through the Friedmann equations.

The structure of this article is the following: In Section 2, we briefly summarize the most significant available constraints about PBH abundances. In Section 3, we analyse the energy exchange between a black hole and a radiation bath. Then, in Section 4, we extend this analysis to a black hole population interacting with radiation in a cosmological background. We present the results in Section 5 and the conclusions and final remarks in Section 6.

2 PBH ABUNDANCES

The constraints on the initial abundances of PBHs are generally expressed in terms of the fraction of the energy density of the Universe that goes to PBHs at their formation epoch: $\beta = \rho_{\text{PBH}}/\rho_{\text{tot}}$.

The lifetime of a black hole with mass M due to Hawking evaporation can be estimated as (Hawking 1975)

$$\tau_{\text{life}} \sim 10^{10} \left(\frac{M}{10^{15} \text{g}} \right) \text{yr}. \quad (2)$$

As described in Carr et al. (2010), this implies that (i) PBHs formed with mass of order of 10^{15}g should be evaporating at the present epoch producing gamma rays, positrons and antiprotons that contribute to the diffuse gamma-ray background and the cosmic-ray flux (e.g. Wright 1996; Carr et al. 2016a). (ii) PBHs with initial mass $M < 10^{15} \text{g}$ are already evaporated; however, their existence could have affected different processes in the early Universe. Ones that evaporated within the first second after the big bang could have generated most of the entropy of the Universe (e.g. Zeldovich & Starobinskii 1976) or altered the baryogenesis (Dolgov, Naselsky & Novikov 2000; Bugaev, Elbakidze & Konishche 2003) and the big

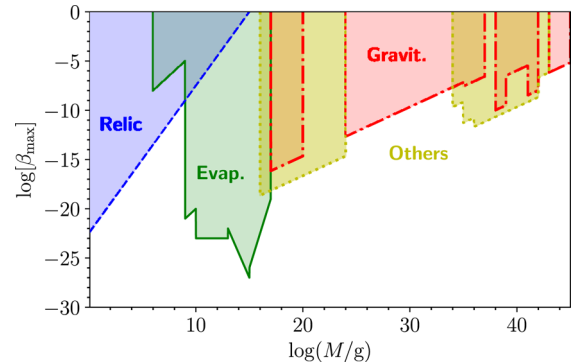


Figure 1. Most relevant constraints on the initial fraction of the energy density of the universe in PBHs for monoenergetic IMFs (adapted from Green 2015). We distinguish in colours the different kind of constraints. The constraints derived from relic particle studies are shown in blue, those related to evaporation effects (e.g. gamma-ray production) in green, those related to gravitational effects (e.g. lensing or dynamical effects) in red and other constraints, as those related to neutron capture or gravitational waves, in yellow. The coloured areas are the forbidden regions of the parameter $\beta = \rho_{\text{PBH}}/\rho_{\text{tot}}$.

bang nucleosynthesis (e.g. Zeldovich et al. 1977; Vainer et al. 1978). These PBHs can also evaporate into neutrinos, hadrons and other massive particles or leave behind Planck-mass relic particles contributing to the cold dark matter (CDM) (e.g. MacGibbon 1987; Barrow et al. 1992; Lemoine 2000; Bugaev & Konishchev 2002; Alexander & Mészáros 2007). (iii) PBHs with $M > 10^{15} \text{g}$ have lifetimes longer than the age of the Universe, and hence they would still exist and would be detectable by their gravitational effects. Indeed, given the negative results obtained so far in the search for particle dark matter (in particular, weakly interacting massive particles, Akerib et al. 2016), PBHs have become interesting CDM candidates (see, e.g. Chapline 1975; Carr et al. 2016b). In addition, these PBHs can interact with other astrophysical objects in several ways; for example, they might be captured by a neutron star and the star being accreted (Capela, Pshirkov & Tinyakov 2013) or they could have played a role as seeds of supermassive black holes in the centre of galaxies. Recently, they have been also proposed as sources of gravitational wave events (García-Bellido 2017).

Since these effects are not observed, constraints on the PBH abundances are imposed in accordance with the sensitivity of current observations. Fig. 1 shows the most updated limits. The constraints on black hole relics are the only ones that can limit the quantity of less massive PBHs formed. It is important to remark, as was done by Carr et al. (2017) and Kühnel & Freese (2017), that the constraints in Fig. 1 are derived considering monoenergetic PBH populations. The case of PBHs with an extended mass function is quite different and depends on the particular form of the IMF. Following Carr et al. (2017), we summarize the main aspects of this treatment in Appendix.

3 BLACK HOLE MASS VARIATION

A Schwarzschild black hole of mass M emits particles with a black-body spectrum at a temperature (Hawking 1975)

$$T_{\text{BH}} = \frac{\hbar c^3}{8\pi k_{\text{B}} G M}, \quad (3)$$

where \hbar is the reduced Planck constant, k_{B} is the Boltzmann constant and G is the gravitational constant. This emission produces a

mass-loss rate given by

$$\left. \frac{\partial M}{\partial t} \right|_{\text{em}} = \frac{1}{c^2} \Sigma_{\text{BH}} h_* \sigma T_{\text{BH}}^4, \quad (4)$$

where σ is the Stefan–Boltzmann constant, c is the speed of light in vacuum, h_* is the number of particle species available for the black hole to evaporate into and Σ_{BH} is the effective area at which particles escape from the hole. Strictly speaking, h_* depends on the temperature and, consequently, on the black hole mass; nevertheless, this dependency is discrete and h_* can be considered constant for fixed mass ranges. Equation (4) can be written as

$$\left. \frac{\partial M}{\partial t} \right|_{\text{em}} = -\frac{A(M)}{M^2}, \quad (5)$$

where $A(M) = 5.3 \times 10^{25} \text{ g}^3 \text{ s}^{-1}$ for black holes with masses $> 10^{17} \text{ g}$ and $A(M) \geq 7.8 \times 10^{26} \text{ g}^3 \text{ s}^{-1}$ for black holes with masses $\leq 10^{15} \text{ g}$ (MacGibbon & Carr 1991).

In addition to the Hawking emission, black holes immersed in a thermal bath accrete particles at a rate given by (Zel'dovich & Novikov 1966)

$$\left. \frac{\partial M}{\partial t} \right|_{\text{acc}} = \frac{27\pi G^2}{c^5} \rho_{\text{R}} M^2, \quad (6)$$

where ρ_{R} is the energy density of radiation measured far away from the hole. Combining the two effects, we obtain the complete equation for the mass variation rate

$$\frac{dM}{dt} = \frac{27\pi G^2}{c^5} \rho_{\text{R}} M^2 - \frac{1}{c^2} \Sigma_{\text{BH}} h_* \sigma T_{\text{BH}}^4. \quad (7)$$

4 COSMIC EVOLUTION

4.1 Two-fluid cosmology

We consider a cosmic fluid immersed in an FLRW space–time, $(\Sigma \times \mathbb{R}, g_{\mu\nu}^{\text{FLRW}})$, where Σ is a set of space-like hypersurfaces and $g_{\mu\nu}^{\text{FLRW}}$ is such that

$$ds^2 = -dt^2 + R(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right]. \quad (8)$$

Here, $R(t)$ is the scalefactor of the metric and k is the curvature parameter. We assume that the fluid is composed of two perfect components, A and B ; hence, its energy-momentum tensor is

$$T^{\mu\nu} = T_{(A)}^{\mu\nu} + T_{(B)}^{\mu\nu}, \quad (9)$$

where

$$T_{(i)}^{\mu\nu} = \frac{1}{c^2} [\rho_{(i)} + p_{(i)}] u^\mu u^\nu - p_{(i)} g^{\mu\nu}, \quad (i = A, B), \quad (10)$$

and $\rho_{(i)}$ and $p_{(i)}$ denote the energy density and the pressure of the fluid-component i , respectively. We assume that both components have the same four-velocity u^μ but their equation of state can be different:

$$p_{(i)} = w_{(i)} \rho_{(i)}, \quad (i = A, B). \quad (11)$$

If the two fluid-components interact, only the total energy-momentum tensor is conserved. Thus,

$$\nabla_\mu T^{\mu\nu} = 0 \implies \nabla_\mu T_{(A)}^{\mu\nu} = -\nabla_\mu T_{(B)}^{\mu\nu}. \quad (12)$$

We denote Q the rate of energy exchange caused by the interaction, and we define the normalized scalefactor $a(t) = R(t)/R(t_0)$, where t_0 is an arbitrary cosmic time (see Section 5 for the specific choice of this value). Then, by adding one of the Friedmann equations to the

only non-trivial component of equation (12), we obtain a system of three differential equations for $\rho_{(A)}$, $\rho_{(B)}$ and a :

$$\begin{aligned} \dot{\rho}_{(A)} + 3 \left(\frac{\dot{a}}{a} \right) [1 + w_{(A)}] \rho_{(A)} &= Q, \\ \dot{\rho}_{(B)} + 3 \left(\frac{\dot{a}}{a} \right) [1 + w_{(B)}] \rho_{(B)} &= -Q, \\ \left(\frac{\dot{a}}{a} \right)^2 - \frac{8\pi G}{3c^2} [\rho_{(A)} + \rho_{(B)}] + \frac{c^2 k}{[R(t_0)a]^2} &= 0. \end{aligned} \quad (13)$$

The interaction term Q depends on the specific characteristics of the system. In what follows, we apply this two-fluid formalism to a cosmological model of the early Universe in which the two fluids are radiation and a PBH gas.

4.2 Early universe with PBHs

Let us consider a relativistic thermal plasma characterized by its equilibrium temperature T_{R} , which sets the other relevant thermodynamic quantities (energy density ρ_{R} , pressure p_{R} and entropy density s_{R}) through the following relations:

$$\begin{aligned} \rho_{\text{R}} &= g_* \frac{(k_{\text{B}} T_{\text{R}})^4}{(\hbar c)^3}, \\ p_{\text{R}} &= \frac{1}{3} \rho_{\text{R}} = g_* \frac{(k_{\text{B}} T_{\text{R}})^4}{3(\hbar c)^3}, \\ s_{\text{R}} &= \frac{\rho_{\text{R}} + p_{\text{R}}}{T_{\text{R}}} = \frac{4}{3} \frac{g_* k_{\text{B}}^4}{(\hbar c)^3} T_{\text{R}}^3. \end{aligned} \quad (14)$$

Here, g_* takes into account the contribution of the different species of relativistic particles.

Let us also consider a PBH component modelled as a dust-like perfect fluid ($p_{\text{PBH}} = 0$) whose constituents are Schwarzschild black holes. These may have different masses and therefore are characterized by their IMF, $N_0(m)$, which evolves with time due to two processes: the expansion of the universe and the energy exchange of each PBH with the radiation. If $N(t; m)$ denotes the mass function at time t , the PBH energy density is

$$\rho_{\text{PBH}}(t) = \int_{M_{\text{min}}}^{M_{\text{max}}} N(t; m) E(m) dm, \quad (15)$$

where M_{min} and M_{max} are the minimum and maximum mass of the black holes and $E(m) = mc^2$ is the energy of a Schwarzschild black hole of mass m . In a similar way, the entropy density can be calculated as

$$s_{\text{PBH}}(t) = \int_{M_{\text{min}}}^{M_{\text{max}}} N(t; m) S(m) dm, \quad (16)$$

where $S(m) = 4\pi k_{\text{B}} G m^2 / \hbar c$ is the Bekenstein–Hawking entropy of a Schwarzschild black hole of mass m .

In order to obtain an expression for the Q -term, we must sum the effects of the interaction of each black hole with the radiation. If the mass of a black hole evolves from m at time t to $m + dm$ at time $t + dt$, then

$$N(t; m) = N(t + dt; m + dm), \quad (17)$$

and this implies

$$\left. \frac{\partial N(t; m)}{\partial t} \right|_{\text{int}} = \frac{\partial N(t; m)}{\partial m} \frac{dm}{dt}, \quad (18)$$

where the mass variation rate is given by equation (7). The Q -term results

$$\begin{aligned} Q &= \left. \frac{\partial \rho_{\text{PBH}}}{\partial t} \right|_{\text{int}} = \int_{M_{\text{min}}}^{M_{\text{max}}} \left. \frac{\partial N(t; m)}{\partial t} \right|_{\text{int}} mc^2 dm \\ &= \int_{M_{\text{min}}}^{M_{\text{max}}} \frac{\partial N(t; m)}{\partial m} \frac{dm}{dt} mc^2 dm. \end{aligned} \quad (19)$$

Finally, we consider that the space–time has negligible curvature (which is a very reasonable assumption in the early Universe) and we set $k = 0$. The system of equations (13) becomes

$$\begin{aligned} \dot{\rho}_{\text{R}} + 4 \frac{\dot{a}}{a} \rho_{\text{R}} &= - \int_{M_{\text{min}}}^{M_{\text{max}}} \frac{\partial N(t; m)}{\partial m} \frac{dm}{dt} mc^2 dm, \\ \dot{\rho}_{\text{PBH}} + 3 \frac{\dot{a}}{a} \rho_{\text{PBH}} &= \int_{M_{\text{min}}}^{M_{\text{max}}} \frac{\partial N(t; m)}{\partial m} \frac{dm}{dt} mc^2 dm, \\ \left(\frac{\dot{a}}{a} \right)^2 &= \frac{8\pi G}{3c^2} (\rho_{\text{R}} + \rho_{\text{PBH}}). \end{aligned} \quad (20)$$

Now we have an integro-differential equation system for the functions $N(t; m)$, $\rho_{\text{R}}(t)$ and $a(t)$. In what follows, we separate our analysis into narrow IMFs and extended IMFs.

4.2.1 Narrow IMF

If a PBH population is formed on a short time-scale, for example from a phase-transition, the IMF is typically narrow and centred on a particular mass (Barrow et al. 1992). We study the simplest narrow IMF, namely a Dirac delta function. We assume that the PBHs form at time t_{form} with a mass $M_{\text{form}} \sim M_{\text{H}}(t_{\text{form}})$. Thus,

$$N_0(m) \equiv A \delta(m - M_{\text{form}}), \quad (21)$$

where A is a normalization constant and can be related to the original fraction of the energy density of the universe that goes to PBHs. The initial energy and entropy densities of the black holes are

$$\rho_{\text{PBH}}(t = t_{\text{form}}) = A M_{\text{form}} c^2, \quad (22)$$

$$s_{\text{PBH}}(t = t_{\text{form}}) = A S(M_{\text{form}}). \quad (23)$$

In this scenario, all black holes evolve in the same manner and at each time t they have the same mass $M_{\text{PBH}}(t)$. Therefore, we can determine the evolution of the whole population by studying one representative PBH. Under this simplification, the set of equations (20) becomes

$$\begin{aligned} \dot{\rho}_{\text{R}} + 4 \frac{\dot{a}}{a} \rho_{\text{R}} &= - A c^2 \frac{dM_{\text{PBH}}}{dt}, \\ \dot{\rho}_{\text{PBH}} + 3 \frac{\dot{a}}{a} \rho_{\text{PBH}} &= A c^2 \frac{dM_{\text{PBH}}}{dt}, \\ \left(\frac{\dot{a}}{a} \right)^2 &= \frac{8\pi G}{3c^2} (\rho_{\text{R}} + \rho_{\text{PBH}}), \end{aligned} \quad (24)$$

that is a system of linear differential equations for $M_{\text{PBH}}(t)$, $\rho_{\text{R}}(t)$ and $a(t)$.

4.2.2 Extended IMF

In other scenarios, for example those where the formation occurs on a long time-scale, the IMF can be extended and span a wide mass range (Barrow et al. 1991b). We analyse the particular case of a power-law IMF of the form

$$N_0(m) = A m^{-\gamma}, \quad (25)$$

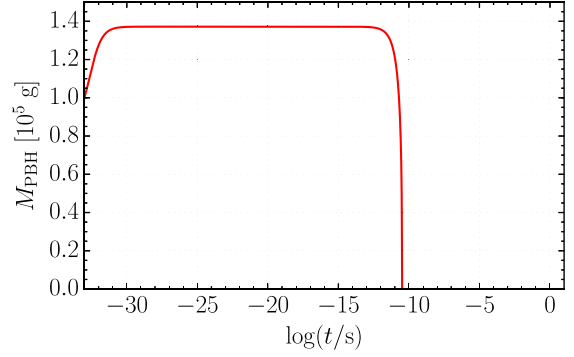


Figure 2. Evolution of a PBH of mass 10^5 g formed at a time $t_{\text{ini}} = 10^{-33}$ s.

where A is a normalization constant and γ is the spectral index, which typically lies in the range of 2–3 (Carr et al. 2010).

Here, we assume that the formation starts at time t_{ini} (the least massive black holes) and ends at time t_{end} (the most massive ones). The form of the mass function $N(t; m)$ varies with time owing to the different rates of evolution of each PBH. In order to solve the equation system, we discretise the function $N(t; m)$ in blocks. Each block evolves as an independent Dirac delta function like the one we previously considered.

5 RESULTS

We set the initial values of the energy densities of both fluids at time t_{ini} , and we normalize the scalefactor such that $a(t_0 = t_{\text{ini}}) = 1$. For the initial radiation temperature, we assume the following expression (e.g. Weinberg 1972):

$$T_{\text{R}}(t_{\text{ini}}) \sim 10^{10} (t_{\text{ini}}/\text{s})^{-1/2} \text{ K}. \quad (26)$$

Then, we set

$$\rho_{\text{PBH}}(t_{\text{ini}}) := \beta \rho_{\text{R}}(t_{\text{ini}}), \quad (27)$$

where $\beta < 1$ is a free parameter of the model, though it is limited by the constraints previously discussed. As the number density of pre-inflation PBHs is negligible after the inflationary epoch, we consider only post-inflationary times: $t_{\text{ini}} = 10^{-33}$ s.

In what follows, we present numerical results for some examples of the two scenarios discussed in Section 4. We solved the equation system (20) using an adapted fourth-order Runge–Kutta method.

5.1 Monoenergetic IMF

Let us consider a monoenergetic PBH population of mass $M_{\text{ini}} = 10^5 \text{ g} \sim M_{\text{H}}(t_{\text{ini}})$. Fig. 2 shows the evolution of one of these black holes. The only free parameter is the ratio of energy densities β . In a first scenario, we do not take into account the rather speculative ‘relic particle’ constraints. Hence, there is no upper limit on β (for the chosen value of the mass) and we set $\beta = 10^{-3}$. Fig. 3 shows the cosmic evolution for this scenario. At the beginning, radiation dominates and the scalefactor evolves as $a(t) \propto t^{1/2}$. As the PBH component dilute slower than radiation, it starts to dominate at some later time, and during a period the universe is PBH-dominated (dust-dominated) and $a(t) \propto t^{2/3}$. All black holes evaporate on a time-scale of about 10^{-10} s yielding their energy to radiation, and the universe becomes radiation-dominated again; however, the PBH population produced an increase in the scalefactor of about two orders of magnitude.

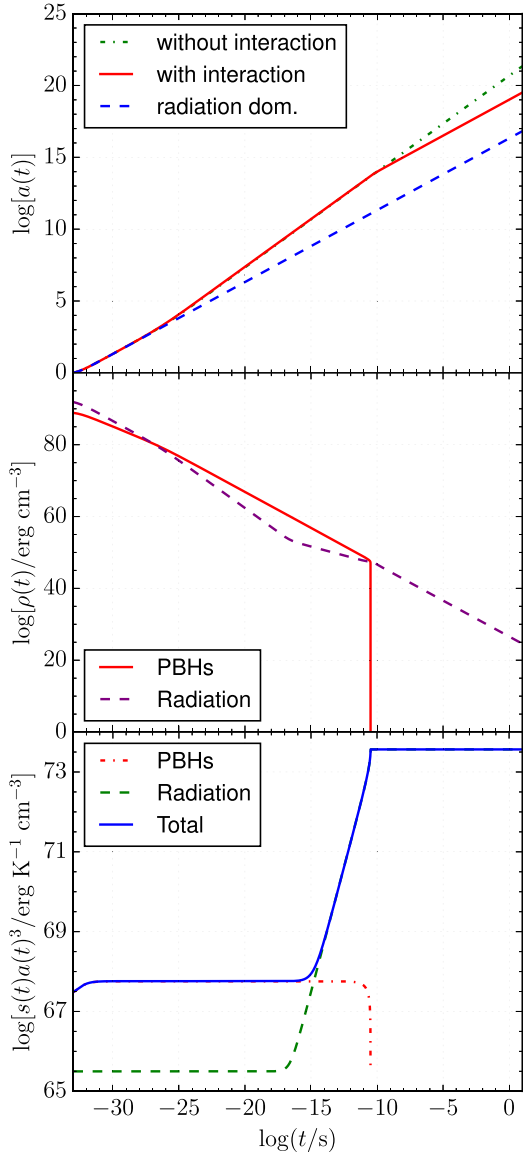


Figure 3. Cosmic evolution for a two-fluid model considering radiation and a monoenergetic population of PBHs with mass $M_{\text{PBH}} = 10^5$ g and $\beta = 10^{-3}$. Top panel shows the scalefactor evolution for this system (with interaction), for a radiation-dominated universe (radiation), and for a universe in which black holes do not evaporate and behave exactly like dust (without interaction). Central panel shows the energy densities of both fluids and the bottom panel shows the entropy per unit of comoving volume of both fluids as well as the total one.

The entropy in a comoving volume, $s(t)a(t)^3$, also increases during the whole evolution; this is driven by the accretion at early times and by the evaporation at the end of the evolution. The latter is the most significant process and produces an increase in entropy by a factor of $\sim 10^6$. This scenario is an example of an a priori plausible scenario (provided that the ‘relic particle’ constraint does not apply), which presents significant modifications compared with the standard radiation-dominated evolution.

If we accept the relic constraints, the highest possible value of β for the adopted mass is $\sim 10^{-15}$. This is a similar case to the situation discussed, but with this new value of β no modifications in the cosmic evolution are obtained, as it is shown in Fig. 4.

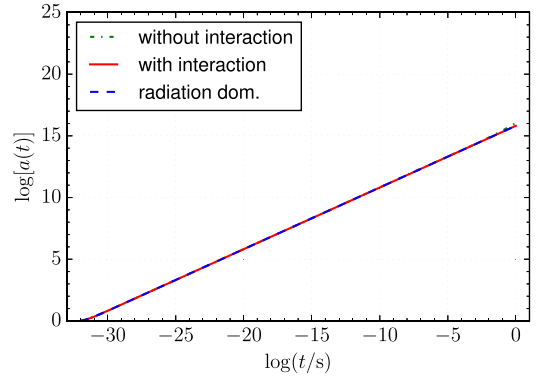


Figure 4. Scalefactor evolution of a two-fluid model considering radiation and a population of PBHs with mass $M_{\text{PBH}} = 10^5$ g and $\beta = 10^{-15}$.

5.2 Power-law IMF

Scenarios with a power-law IMF were studied by Barrow et al. (1991b), who neglected the accretion and considered that all PBHs formed simultaneously. We assume here that a black hole of mass m cannot be formed until the horizon mass exceeds m , and hence the formation must be extended in time. There are three additional free-parameters besides β : the minimum and maximum mass of the distribution, M_{min} y M_{max} and the spectral index γ .

In order to investigate the general behaviour of power-law IMFs, we first ignore the constraints. We choose $M_{\text{min}} = 6.5 \times 10^2 m_{\text{p}}$ ($m_{\text{p}} \simeq 2.18 \times 10^{-5}$ g is the Planck mass), which corresponds to black holes with lifetimes of the order of the initial time ($t_{\text{life}} \sim t_{\text{ini}} = 10^{-33}$ s) and $M_{\text{max}} = 10^6 M_{\odot}$, corresponding to supermassive black holes. We study IMFs with different values of γ in the range of 2–3, and for different values of β . In addition, as we may have large enough PBHs for accretion to be important, we consider scenarios with and without accretion. We fit the scalefactor with power laws: $a \propto t^r$.

In agreement with Barrow et al. (1992), we find scenarios with intermediate evolution between those that are radiation and dust-dominated ($1/2 < r < 2/3$). Significant modifications in the cosmic evolution occur only for γ near 2 (harder spectra) and for higher values of β . Fig. 5 shows the results for the case $\gamma = 2.1$, $\beta = 10^{-1}$. The scalefactor evolves as a power law with $r = 0.544$ independently of whether there is accretion or not. Furthermore, despite accretion temporarily modifies the ratio of energy densities, the dilution of the holes rapidly becomes dominant and the two scenarios converge to the same stationary value of $\beta \sim 0.47$; accretion seems to be negligible even in the most favourable scenario. Finally, the entropy in a comoving volume increases during the whole evolution although less than in one of the monoenergetic cases discussed above, even though here β is two orders of magnitude higher.

A plausible power-law scenario must satisfy the observational constraints. Fig. 6 shows how these constraints apply for this type of distribution as a function of γ . We can see that the harder the spectrum is, the lower the influence of the relic constraints results. The most favourable scenarios are those with hard spectra; for them $\beta \sim 10^{-24}$ both neglecting and including the relic constraints. For these cases, we obtain that PBHs do not cause any significant effect and the scalefactor evolves as in a radiation-dominated universe.

6 CONCLUSIONS AND FINAL REMARKS

We have studied the effects that different PBH populations produce on the evolution of the early Universe. We considered a two-fluid

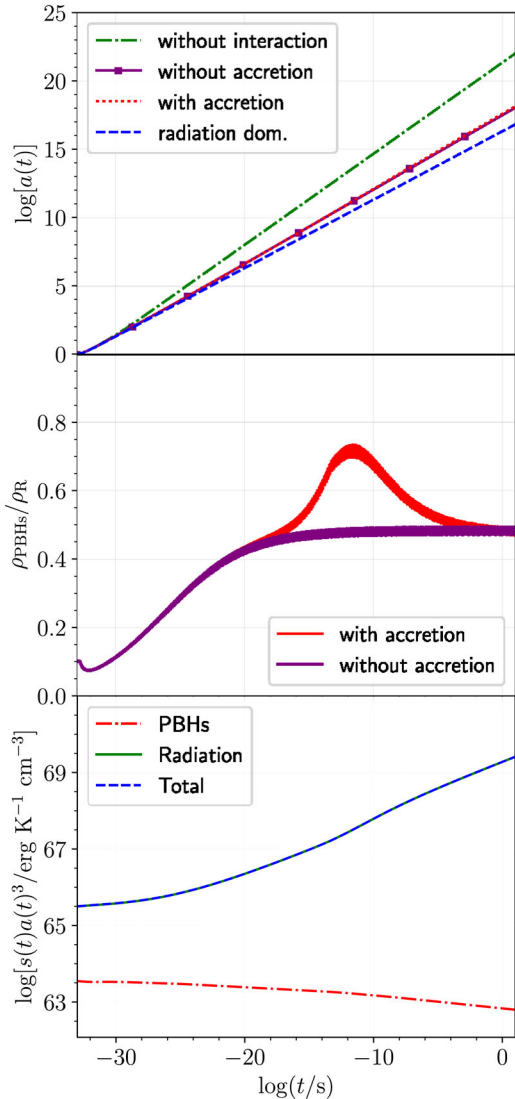


Figure 5. Cosmic evolution for a power-law IMF with spectral index $\gamma = 2.1$, and with masses lying between $M_{\min} = 650m_p$ and $M_{\max} = 10^6 M_{\odot}$, for a value of $\beta = 10^{-1}$. Top panel shows the scale-factor evolution in four cases: with and without accretion, in a radiation-dominated universe, and in a universe with radiation and PBHs but without any interaction between them. Central panel shows the ratio of the fluid energy densities with and without accretion. Bottom panel shows the entropy per unit of comoving volume of both fluids as well as the total one.

cosmological model with radiation and a PBH gas. According to the different possible formation mechanisms, the PBH IMF may be extended or narrow. We have investigated representative cases for these two possible scenarios, namely power-law and monoenergetic IMFs. To characterize the IMFs, we have taken into account the different constraints that exist for PBH abundances. In particular, we have distinguished the ‘relic constraints’ from the others, since they are too conjectural.

Monoenergetic populations of small PBHs produce significant modifications in the cosmic evolution (changes in the scalefactor and generation of entropy) provided that the PBH energy density is high enough. The latter condition requires to discard the ‘relic constraints’. If instead we take them as valid, no effects are produced.

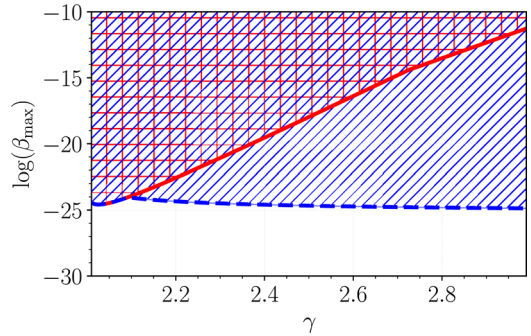


Figure 6. Combined constraints on the initial fraction of the energy density of the universe in PBHs for power-law IMFs, as a function of the spectral index γ . The minimum mass is $M_{\min} = 650m_p$. The red line is plotted discarding the relic constraints, whereas the blue dashed-line considers them. The hatched areas are the forbidden regions of the parameter β . In Appendix , we show a more general case.

The behaviour of power-law scenarios is different. We discussed that the constraint treatment depends on the IMF form. For the analysed cases, we found cosmic evolution modifications only if we omit the constraints. In addition, we investigated the importance of accretion in these scenarios finding that it plays no significant role. Situations where all constraints are satisfied do not present any relevant effect.

We conclude that the presence of some particular PBH populations in the early Universe may affect the cosmic evolution. In particular, populations of small PBHs with narrow IMFs are likely to produce these effects. These populations are possible if the relic constraints are not valid. In addition, we found that accretion on to PBHs is not a relevant energy-exchange mechanism in the early Universe, even for the most favourable IMFs.

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APPENDIX: PBH CONSTRAINTS ON EXTENDED MASS FUNCTIONS

Let us consider a PBH population with mass function $N(m; \alpha)$, where $\alpha = \{\alpha_i\}$ are the parameters that characterize the function,

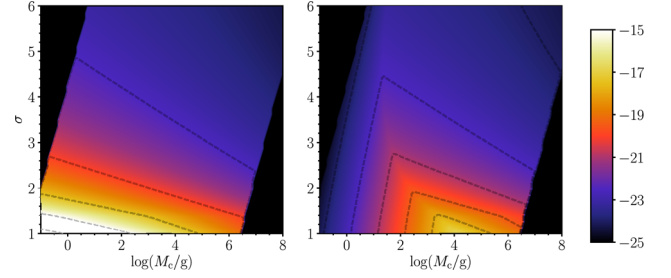


Figure A1. Combined effect of the constraints on the initial fraction of the energy density of the universe in PBHs for power-law IMFs, as a function of the parameters $M_c = M_{\min} e^{1/(\gamma-2)}$ and $\sigma \equiv 1/(\gamma-2)$. The colour map shows $\log(\beta_{\max})$. The dotted lines are contour lines of β_{\max} (i.e. $\beta_{\max} = \text{constant}$) and the regions in black are forbidden combinations of the parameters. The left-hand panel is plotted discarding the relic constraints, whereas in the right-hand panel these are considered.

and let us define

$$\phi(m; \alpha) := \rho_{\text{tot}}^{-1} N(m; \alpha) m c^2, \quad (\text{A1})$$

where ρ_{tot} is the total energy density of the universe and c is the speed of light in vacuum. If $\mathcal{O}[\phi(m; \alpha)]$ denotes an observable depending on the PBH mass function, we can expand it as

$$\begin{aligned} \mathcal{O}[\phi(m; \alpha)] &= \mathcal{O}_0 + \int dm \phi(m; \alpha) K_1(m) \\ &+ \int dm_1 dm_2 \phi(m_1; \alpha) \phi(m_2; \alpha) K_2(m_1, m_2) + \dots, \end{aligned} \quad (\text{A2})$$

where \mathcal{O}_0 is the background contribution and the functions $K_j(m)$ depend on the details of the underlying physics and the nature of the observation. As we considered that PBHs do not interact among themselves, only the first two terms in the right-hand side of equation (A2) need to be considered.

If a measurement imposes an upper bound on the observable,

$$\mathcal{O}[\phi(m; \alpha)] \leq \mathcal{O}_{\text{exp}}, \quad (\text{A3})$$

for a monoenergetic mass function with $m = M_*$,

$$\phi_{\text{mon}}(M_*) \equiv \rho_{\text{tot}}^{-1} A \delta(m - M_*) m c^2, \quad (\text{A4})$$

we have

$$\rho_{\text{tot}}^{-1} A M_* c^2 \leq \frac{\mathcal{O}_{\text{exp}} - \mathcal{O}_0}{K_1(M_*)} \equiv \beta(M_*), \quad (\text{A5})$$

where $\beta(M_*)$ is the upper bound for monoenergetic distributions (see Fig. 1). Combining equations (A2) and (A5) we obtain

$$\int dm \frac{\phi(m; \alpha)}{\beta(m)} \leq 1, \quad (\text{A6})$$

for an arbitrary mass function. If we know $\beta(m)$ and assume the form of the function $\phi(m; \alpha)$, we can integrate equation (A6) over the mass range (m_1, m_2) for which the constraint applies. For given values of the parameters α , this imposes limits on $\beta \equiv \rho_{\text{PBH}}/\rho_{\text{tot}}$. In particular, for a power-law mass function we have

$$\phi(m; M_{\min}, \gamma) = A m^{1-\gamma} c^2, \quad \text{with } m \geq M_{\min}. \quad (\text{A7})$$

If $\gamma > 2$, a minimum mass is strictly necessary for the function not to diverge. Instead, the role of the maximum mass is not important in

this case. Then, for each set of parameters $\{\gamma, M_{\min}\}$ the combined effect of equation (A6) applied to the different constraints impose limits on A and hence on β . It is standard to plot the constraints as a function of the parameters $M_c \equiv M_{\min} e^{1/(\gamma-2)}$ and $\sigma \equiv 1/(\gamma-2)$ instead of the original ones (see Fig. A1).

The procedure discussed in this Appendix is an adaptation of the one presented by Carr et al. (2017).

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