Orbital and spin dynamics of intraband electrons in quantum rings driven by twisted light

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Abstract: We theoretically investigate the effect that twisted light has on the orbital and spin dynamics of electrons in quantum rings possessing sizable Rashba spin-orbit interaction. The system Hamiltonian for such a strongly inhomogeneous light field exhibits terms which induce both spin-conserving and spin-flip processes. We analyze the dynamics in terms of the perturbation introduced by a weak light field on the Rasha electronic states, and describe the effects that the orbital angular momentum as well as the inhomogeneous character of the beam have on the orbital and the spin dynamics.

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1. Introduction

In a recent series of articles we studied the interaction of twisted light (TL) [1] with semiconductor nanostructures, and showed that interesting new effects are produced because of the orbital angular momentum (OAM) and the inhomogeneous character of the TL beam. In particular, we demonstrated that circulating electric currents are generated in interband transitions in semiconductor quantum rings (QR) [2], and that new electronic transitions become optically allowed in semiconductor quantum dots (QD) [3, 4]. Nevertheless, the topic of spin dynamics driven by TL has not been, to the best of our knowledge, addressed so far in the literature. Detailed descriptions of spin dynamics are of the utmost importance in condensed matter physics, from first principles to applications in spintronics. At the same time, the control of nanosystems by optical means is a very active field of research, for it proves to be an efficient, fast technique to manipulate quantum states.

In this article, we report our theoretical predictions on the orbital and the spin dynamics of conduction-band electrons in QR illuminated with TL, when the Rashba spin-orbit interaction (SOI) in the QR is taken into account. By comparison with the canonical case of irradiation with plane waves, we show that a variety of new effects arise, some connected to the orbital angular momentum of the TL beam, and others to its inhomogeneous character.

In Sec. 2 the theoretical model is introduced, the matrix elements of the TL-QR interaction are obtained in Sec. 3, Sec. 4 studies the quantum evolution of the photoexcited QR, and conclusions are presented in Sec. 5.

2. Twisted light and Quantum rings

The system under investigation is a phase-coherent mesoscopic quantum ring of radius a, thickness d, and height h (with $a \gg d,h$), illuminated at normal incidence (z-axis) by THz twisted light radiation. We consider the situation where the QR and the TL symmetry axes coincide, which we believe poses no technological difficulty since large QRs are currently manufactured [5]; otherwise, our previous findings on off-centered beams [4] may serve to clarify experimental results. The electronic states of the QR are described in the envelope-function approximation, and will be given after the Hamiltonian is introduced. While studying spin and orbital dynamics, we will assume that there is one electron occupying initially ($t = 0^-$) a conduction-band QR eigenstate. Experimentally, if phase-coherence is desired, net charge in the conduction-band states is injected via modulation doping, in order to reduce impurity scattering. Electrons can also be promoted to the conduction-band states by means of photo-excitation with ultrashort optical pulses, although in this case their lifetime prior to recombination is of the order of nanoseconds. THz TL radiation will then induce intraband transitions.

The TL beam is represented by its vector potential in cylindrical coordinates (keeping only its transverse components) [6]

$$\begin{aligned} \mathbf{A}(\mathbf{r},t) &= \boldsymbol{\varepsilon}_\sigma F_l(q_r r)\, e^{i(q_z z-\omega t)}\, e^{il\phi} + c.c. \\ &= \mathbf{A}^{(+)}(\mathbf{r},t) + \mathbf{A}^{(-)}(\mathbf{r},t), \end{aligned} \tag{1}$$

with the polarization vectors given by $\boldsymbol{\varepsilon}_\sigma = \hat{x} + \sigma i\hat{y} = e^{\sigma i\phi}(\hat{r} + \sigma i\hat{\phi})$, $\sigma = \pm 1$, and $c.c.$ denoting the complex conjugate. The radial function $F_l(q_r r)$ is left, for the moment, unspecified. Disregarding the longitudinal component of $\mathbf{A}$ is justified in usual experimental conditions, in which $q_z \gg q_r$.

The Hamiltonian of the system of QR plus TL, including the Rashba spin-orbit coupling in the QR [7], is decomposed into

$$H = H_{\mathrm{SOI}} + H_1,$$

Table 1. Notation

$\tan\gamma = -\omega_R/\omega_0$
$\hbar\omega_R = 2\alpha_R/a$
$\hbar\omega_0 = \hbar^2/(m_e^* a^2)$
$x_s = -(1-sw)/2$
$w = \sqrt{1+Q_R^2} = 1/\cos(\gamma)$

with

$$H_{\rm SOI} = \frac{\hat{p}^2}{2m_e^*} + V(\mathbf{r}) + \frac{\alpha_R}{\hbar}\left[\hat{\boldsymbol{\sigma}}\times\hat{\mathbf{p}}\right]_z \tag{2}$$

$$H_1 = -\frac{q}{m_e}\mathbf{A}(\mathbf{r},t)\cdot\hat{\mathbf{p}} - \frac{q\alpha_R}{\hbar}[\hat{\boldsymbol{\sigma}}\times\mathbf{A}(\mathbf{r},t)]_z \tag{3}$$

where $q=-e$ and m_e (m_e^*) are the charge and mass (effective mass) of the electron, $V(\mathbf{r})$ the QR confinement potential, and $\hat{\boldsymbol{\sigma}}$ is the vector of Pauli matrices. The Rashba coupling constant is α_R. The perturbation H_1 introduced by the light beam has been deduced from minimal coupling up to first order in $\mathbf{A}(\mathbf{r},t)$. We leave out the Dresselhaus spin-orbit coupling since, as shown in Ref. [8], in quasi-one-dimensional structures it can be eliminated by an adequate choice of the lateral confinement.

In principle, Eq. (3) should include a Zeeman term coming from the magnetic component of the TL field. A simple calculation of $\nabla\times\mathbf{A}(\mathbf{r},t)$ shows that this magnetic field has transverse as well as longitudinal components, which are proportional to q_z and q_r, respectively. Therefore, both are small compared to the other terms in the Hamiltonian and can be safely neglected.

Eq. (3) exhibits the unfamiliar term $(-q\alpha_R/\hbar)[\hat{\boldsymbol{\sigma}}\times\mathbf{A}(\mathbf{r},t)]_z$ coupling linearly the light electric field to the spin operators via the Rashba-type spin-orbit coupling. Note that the final expression of the Hamiltonian in Ref. [7] does not contain an analogous term. That simplification was possible due to the applicability of the dipole approximation in that work. The Göppert-Mayer transformation, leading to the dipole-moment Hamiltonian, is not possible in our problem due to the inhomogeneous nature of the TL beam. Therefore, we continue our analysis within the Coulomb gauge which leads to the Hamiltonian of Eqs. (2,3).

As has been shown in previous works [9, 7] the Hamiltonian $H_{\rm SOI}$ can be exactly diagonalized, having envelope eigenfunctions $\Phi_{ns}(\mathbf{r}) = \psi_{ns}(\varphi)R(r)Z(z)$ with

$$\psi_{ns}(\varphi) = \frac{1}{\sqrt{2\pi}}e^{i(n+1/2)\varphi}\nu_s(\gamma,\varphi)\,, \tag{4}$$

with the z-projection of the total angular momentum $n+1/2$, angular coordinate φ, and angle-dependent spinor

$$\nu_1(\gamma,\varphi) = \begin{pmatrix} \cos(\gamma/2)e^{-i\varphi/2} \\ \sin(\gamma/2)e^{i\varphi/2} \end{pmatrix} \tag{5}$$

$$\nu_{-1}(\gamma,\varphi) = \begin{pmatrix} -\sin(\gamma/2)e^{-i\varphi/2} \\ \cos(\gamma/2)e^{i\varphi/2} \end{pmatrix}, \tag{6}$$

where γ is the angle between the z-axis and the spin; this angle depends on the quantum number n, but for negligible Zeeman energy, it becomes independent of n: $\tan\gamma = -\omega_R/\omega_0$, with $\hbar\omega_R = 2\alpha_R/a$ and $\hbar\omega_0 = \hbar^2/(m_e^* a^2)$. The energies of the states are

$$\varepsilon_{ns} = \frac{\hbar\omega_0}{2}\left[(n-x_s)^2 - \frac{Q_R^2}{4}\right], \tag{7}$$

with $x_s = -(1-sw)/2$, $w = \sqrt{1+Q_R^2} = 1/\cos(\gamma)$, and $s = \pm 1$ for respectively spin up and down in the (φ-dependent) local frame. (The notation is summarized in Table 1.)

3. Interaction between twisted light and quantum rings

We now obtain the matrix elements of the light-matter interaction given by Eq. (3), in the case of TL beams (Eq. (1)) applied on the QR described in the previous Section. The matrix elements are thus calculated in the basis set of the QR states, Eq. (4), and we will treat separately the two terms of $H_1 = H_{11} + H_{12}$ given in Eq. (3). In Sec. 4 we will employ the matrix elements obtained here in order to analyze the orbital and spin dynamics of the QR in time-dependent perturbation theory, considering H_1 as a perturbation to $H_{\rm SOI}$.

3.1. *Hamiltonian* $H_{11} = -(q/m_e)\mathbf{A}(\mathbf{r},t)\cdot\hat{\mathbf{p}}$

As customary, we separate the Hamiltonian into positive and negative parts

$$\begin{aligned} H_{11} &= H_{11}^{(+)} + H_{11}^{(-)} \\ &= -\frac{q}{m_e}\left[\mathbf{A}^{(+)}(\mathbf{r},t) + \mathbf{A}^{(-)}(\mathbf{r},t)\right]\cdot\hat{\mathbf{p}}. \end{aligned} \tag{8}$$

Let us calculate the matrix element $\langle n's'|H_{11}^{(+)}|ns\rangle$ of the positive term, between initial $|ns\rangle$ and final $\langle n's'|$ states, where $\langle \mathbf{r}|ns\rangle = \Phi_{ns}(\mathbf{r})u_\lambda(\mathbf{r})$, $u_\lambda(\mathbf{r})$ is the microscopic (with lattice periodicity) wave-function. A series of simplifications are possible thanks to the assumption that all processes occur in the same subband: *i*) $\hat{\mathbf{p}}$ acting on the microscopic wave-function yields a vanishing matrix element; *ii*) $\hat{\mathbf{p}}$ acting on the envelope wave functions $Z(z)$ and $R(r)$ would induce transitions between different z/r-subbands, and thus are disregarded. In addition, since the QR is thin, the smooth function $F_l(q_r r)$ can be taken as constant at the value of $r = a$ and pulled out of the matrix element. Finally, we separate the integral over the whole QR into an integral on the cell and a sum (that can be taken as an integral) over all cells [10]. After these simplifications, we are left with the element

$$\begin{aligned} \langle n's'|H_{11}^{(+)}|ns\rangle &= -\sigma\frac{1}{\sqrt{2}}\frac{\hbar q}{m_e}e^{iq_z z_0}F_l(q_r a)\,e^{-i\omega t} \\ &\times \int_V d^3\mathbf{r}\,\Phi^*_{n's'}(\mathbf{r})\,e^{i(l+\sigma)\varphi}\left[\frac{1}{r}\partial_\varphi\Phi_{ns}(\mathbf{r})\right]. \end{aligned} \tag{9}$$

Using Eq. (4)

$$\partial_\varphi\psi_{ns}(\varphi) = i(n+1/2)\psi_{ns}(\varphi) - \frac{is}{2\sqrt{2\pi}}e^{i(n+1/2)\varphi}\,\nu_s(-\gamma,\varphi). \tag{10}$$

Then,

$$\langle n's'|H_{11}^{(+)}|ns\rangle = \xi_\sigma\,e^{-i\omega t}\,\delta_{l+\sigma,n'-n}[\delta_{s,s'}(n+1/2-s\cos\gamma) + \delta_{s,-s'}\sin\gamma], \tag{11}$$

where $\xi_\sigma = -\frac{i\sigma}{\sqrt{2}}\frac{\hbar q}{m_e a}e^{iq_z z_0}F_l(q_r a)$. The product of spinors does not depend on φ, and was pulled out of the integral. For the negative part $H_{11}^{(-)}$ we simply use $\langle n's'|H_{11}^{(-)}|ns\rangle = \langle ns|H_{11}^{(+)}|n's'\rangle^*$. We see that the Hamiltonian H_{11}, though not a spin-orbit interaction, produces spin flips due to the last term of Eq. (11). This is possible because H_{11} acts upon eigenstates of the Rashba Hamiltonian $H_{\rm SOI}$, which are not eigenstates of $\hat{\sigma}_z$. In the case of weak Rashba effect, the spin-flip term is proportional to the angle $\gamma \propto \alpha_R$; for vanishing spin-orbit coupling, we recover the spin-conserving interaction.

3.2. *Hamiltonian* $H_{12} = -(q\alpha_R/\hbar)[\hat{\boldsymbol{\sigma}} \times \mathbf{A}(\mathbf{r},t)]_z$

As mentioned previously,

$$H_{12} \quad = \quad -\frac{q\alpha_R}{\hbar}[\hat{\boldsymbol{\sigma}} \times \mathbf{A}(\mathbf{r},t)]_z \tag{12}$$

appears as a consequence of the inhomogeneous nature of the light field and, in contrast to H_{11}, is a spin-light coupling which is linear in the electric field strength. This dependence stems from the functional form of the Rashba-type spin-orbit coupling.

Again, for the positive part of the vector potential

$$\begin{aligned}\langle n's'|H_{12}^{(+)}|ns\rangle \quad &= \quad -\frac{q\alpha_R}{2\pi\sqrt{2\hbar}}e^{iq_z z_0}F_l(q_r a)\,e^{-i\omega t} \\ &\times \int_V d\varphi\, e^{-i(n'-n-l)\varphi} v_{s'}^{\dagger}(\gamma,\varphi)(\sigma i\hat{\sigma}_x - \hat{\sigma}_y)v_s(\gamma,\varphi)\,.\end{aligned}$$

The products $v_{s'}^{\dagger}(\gamma,\varphi)(\sigma i\hat{\sigma}_x - \hat{\sigma}_y)v_s(\gamma,\varphi)$ can be evaluated, and brought into the matrix form

$$i\sigma e^{i\sigma\varphi}\begin{pmatrix} \sin\gamma & \cos\gamma+\sigma \\ \cos\gamma-\sigma & -\sin\gamma \end{pmatrix} = i\sigma e^{i\sigma\varphi}M_\sigma\,. \tag{13}$$

Note how the polarization of the light couples to the spin degree of freedom, through the off-diagonal terms. With these expressions, the matrix element reads

$$\langle n's'|H_{12}^{(+)}|ns\rangle \quad = \quad \eta_\sigma\, e^{-i\omega t}\,\delta_{n'-n,l+\sigma}\,M_{\sigma s's}\,, \tag{14}$$

where $\eta_\sigma = -i\sigma\frac{q\alpha_R}{\hbar}e^{iq_z z_0}F_l(q_r a)$.

3.3. *The total perturbation* H_1

A compact matrix form of the total perturbation, in a representation where $v_1^{\dagger} = (1,0)$ and $v_{-1}^{\dagger} = (0,1)$, is

$$\begin{aligned}H_{1,n'n} \quad &= \quad e^{-i\omega t}\,\delta_{n'-n,l+\sigma} \\ &\times\left[\xi_\sigma\begin{pmatrix} n+\frac{1}{2}-\cos\gamma & \sin\gamma \\ \sin\gamma & n+\frac{1}{2}+\cos\gamma \end{pmatrix} + \eta_\sigma\begin{pmatrix} \sin\gamma & \cos\gamma+\sigma \\ \cos\gamma-\sigma & -\sin\gamma \end{pmatrix}\right] + H.c.\,,\end{aligned} \tag{15}$$

where the $H.c.$ implies not only the conjugate and transposition of the spin matrix, but also the transposition $n \leftrightarrow n'$. We remind the reader that the terms proportional to ξ_σ (η_σ) come from H_{11} (H_{12}).

4. Evolution of single-particle states

We use standard time-dependent perturbation theory, for the case of a harmonic perturbation. The general formula for the wave function is

$$\Psi(\mathbf{r},t) = \sum a_{ns}(t)e^{-i\,\varepsilon_{ns}t/\hbar}\psi_{ns}(\varphi) \tag{16}$$

where the coefficients are given by

$$\begin{aligned}a_{ns}^{(1)}(t) \quad &= \quad \frac{1}{\hbar}\langle ns|H_1^{(+)}(0)\,|n_0 s_0\rangle\frac{1-e^{i(\omega_{fi}-\omega)t}}{\omega_{fi}-\omega} \\ &+\frac{1}{\hbar}\langle ns|H_1^{(-)}(0)\,|n_0 s_0\rangle\frac{1-e^{i(\omega_{fi}+\omega)t}}{\omega_{fi}+\omega}\end{aligned} \tag{17}$$

where $\{n_0, s_0\}$ are the quantum numbers of the initial state, and $\hbar\omega_{fi} = \varepsilon_{ns} - \varepsilon_{n_0 s_0}$.

In contrast to *inter*-band transitions, where the Rotating Wave Approximation (RWA) is usually applied in expressions such as Eq. (17)—e.g. by neglecting the term with $\omega_{fi} + \omega$ for absorption—, in *intra*-band transitions this approximation is not justified. In fact, the resonant $|\omega_{fi}| - \omega$ and non-resonant $|\omega_{fi}| + \omega$ terms are of the same order of magnitude, and should both in principle be included.

Some comments on the topic of gauge invariance as related to twisted light are in order. As previously mentioned, we work in the Coulomb gauge, where the vector potential appears as the main quantity, in contrast to other treatments of light-matter interaction based on the light's electric and magnetic fields. In the standard—and simplest—case of plane waves, the Göppert-Mayer transformation on a fully homogeneous field yields a Hamiltonian whose $\mathbf{p}^2/2m$ is gauge invariant, thus representing a physical quantity, i.e. the kinetic energy; in contrast, the same term in the Coulomb gauge is not, by itself, gauge invariant. As a consequence, for example, the eigenvectors of $\mathbf{p}^2/2m + V(\mathbf{r})$, though the same mathematical functions, do not have the same meaning in both gauges. In general, the correspondence between the predictions obtained in both gauges is achieved via a transformation of the wave function. In the special case of the calculation of transition probabilities, these coincide in both gauges without the need of a transformation of the wave function if the vector potential is zero at the initial and final times, as explicitly demonstrated by Lamb *et al.* [11]. In addition, the discrepancies between the results of a $\mathbf{E}(t)\cdot\mathbf{d}$ and a $\mathbf{A}(t)\cdot\mathbf{p}$ approach are reported to be smaller the closer to resonance excitation one is [11, 12]. This suggests that consideration of the gauge invariance problem is more important in our present work, where we deal with intraband excitation. Furthermore, the case of twisted light is a more delicate one, since the inhomogeneous character of the beam precludes our use of the Göppert-Mayer transformation. In any case, our results, obtained within a given gauge, are perfectly valid even though they may require further work in order to be used to interpret particular experimental situations.

4.1. General considerations

For arbitrary OAM l and polarization σ of the light, Eq. (15) shows that transitions to nearby and distant states are possible. We mention that a particular situation happens for the value $l = -\sigma$: the *total* angular momentum of the electron is unchanged by the light.

In general, we can say that in this system there will be two different time scales present in the quantum evolution: one associated with the evolution of the orbital, and another one with the spin, degrees of freedom. In the limit of zero Rashba SOI, there would be no spin evolution. By the same token, since the SOI is in general a weak interaction, the spin evolution will be slow compared to the orbital one.

The rate of spin conversion achieved by TL irradiation is not simply proportional to the OAM of the beam, but it is rather related to the beam function $F_l(q_r r)$ for the radial profile via the constants ξ and η. The orbital motion is dictated by the constant ξ, while the spin evolution (spin-flip) by both constants ξ and η.

The case of plane waves can be deduced from our formalism by taking $l = 0$. Although in this case a gauge transformation could be made towards a $\mathbf{E}(t)\cdot\mathbf{d}$ Hamiltonian, our present Hamiltonian can still be used. Then, Eq. (15) shows that spin flips occur even when $l = 0$ due to the presence of the Rashba SOI.

In what follows we consider in more detail the evolution under small spin-orbit coupling, since it is the usual situation in real materials.

4.2. Small spin-orbit coupling

The condition $\omega_R \ll \omega_0$ allows for the simplification of the Hamiltonian Eq. (15) to

$$\begin{aligned} H_{1,n'n} &= e^{-i\omega t}\delta_{n'-n,l+\sigma} \\ &\times\left[\xi_\sigma\begin{pmatrix} n-\frac{1}{2} & \gamma \\ \gamma & n+\frac{3}{2}\end{pmatrix}+\eta_\sigma\begin{pmatrix} 0 & 1+\sigma \\ 1-\sigma & 0\end{pmatrix}\right]+H.c. \end{aligned} \tag{18}$$

We exemplify by studying the evolution of a particle initially in state $|n_0,1\rangle$. A TL field, having $l=1$ and $\sigma=1$, is turned on at time $t=0^+$ and turned off at time $t=T$. Then, the wave function is that of Eq. (16) with coefficients

$$\begin{aligned} a^{(1)}_{n_0-2,-1}(T) &= \frac{1-e^{i(\omega_{fi}+\omega)T}}{\hbar(\omega_{fi}+\omega)}(\xi_1^*\gamma+2\eta_1^*) \\ a^{(1)}_{n_0+2,-1}(T) &= \frac{1-e^{i(\omega_{fi}-\omega)T}}{\hbar(\omega_{fi}-\omega)}\xi_1\gamma \\ a^{(1)}_{n_0+2,1}(T) &= \frac{1-e^{i(\omega_{fi}-\omega)T}}{\hbar(\omega_{fi}-\omega)}\xi_1(n_0-1/2) \\ a^{(1)}_{n_0-2,1}(T) &= \frac{1-e^{i(\omega_{fi}+\omega)T}}{\hbar(\omega_{fi}+\omega)}\xi_1^*(n_0-5/2)\,, \end{aligned} \tag{19}$$

where ω_{fi} is understood as having the right indices for each case; while in all cases $i=\{n_0,1\}$, for example, for the third line of Eq. (19), $f=\{n_0+2,1\}$.

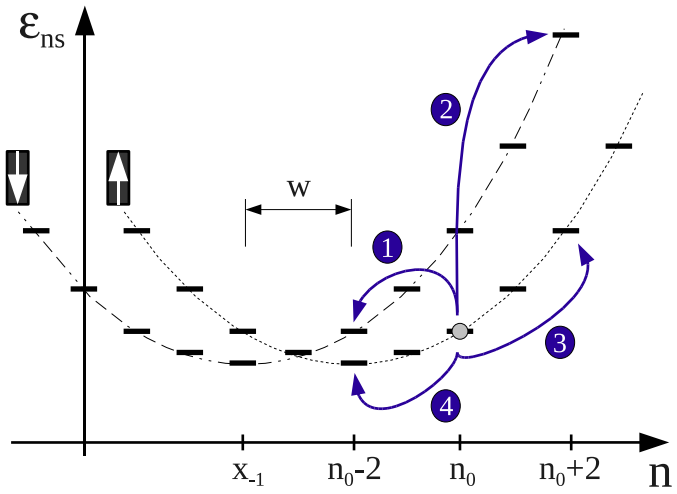

Fig. 1. Pictorial representation of the electronic bands and a transition induced by TL having OAM $l=1$ and $\sigma=1$. According to time-dependent perturbation theory: an electron initially in the state $\{n_0,\uparrow\}$ evolves into a superposition of neighboring states having the same and the opposite spin states. Transitions, indicated by enclosed numbers (blue), correspond to the coefficients: (1) $a^{(1)}_{n_0-2,-1}(t)$, (2) $a^{(1)}_{n_0+2,-1}(t)$, (3) $a^{(1)}_{n_0+2,1}(t)$, (4) $a^{(1)}_{n_0-2,1}(t)$, of Eqs. (19).

A pictorial representation of the process is given in Fig. 1. The initial state evolves into a superposition of neighboring states having the same and the opposite spin states and differing in their total angular momentum by $l+\sigma=2$. The perturbation H_{11} is responsible for spin-conserving and spin-flip terms, while H_{12} causes only spin flips. As seen from Eq. (18), the ratio $\eta/(\xi\gamma)=1/\sqrt{2}$ determines the relative contribution for spin conversion between H_{11} and H_{12}. On the other hand, the rate of change of orbital and spin degrees of freedom can be estimated from the corresponding matrix element of the interaction Hamiltonian. The rates are then estimated by $\Gamma_s=\xi\gamma/\hbar$ for the spin and $\Gamma_o=\xi/\hbar$ for the orbital degrees of freedom, as can also be easily seen from the expressions for $a^{(1)}_{n_0+2,-1}(T)$ and $a^{(1)}_{n_0+2,1}(T)$ in Eq. (19). In the case of a GaAs QR of radius $a=10^{-6}$ m having $\alpha_R=10^{-14}$ V m and pulsed laser parameters typical for experiments: power 10^6 J Hz, repetition rate 10^6 Hz and wavelength 10^{-5} m, we obtain $\Gamma_s=0.1\,\mathrm{ps}^{-1}$ and $\Gamma_o=0.5\,\mathrm{ps}^{-1}$.

One can notice that, even without applying the RWA, only one of the terms in Eq. (17) survives for each final state f. This is due to the fact that we keep track of the selection rule for the momentum conservation (encoded in the matrix element of the interaction Hamiltonian), which is usually missing in the standard treatment of plane-waves excitation. In fact, the four terms are in Eq. (19) correspond to resonant processes, since $\omega_{fi}<0$ for transitions # 1 and # 4. Moreover, the processes are real, in the sense that an electronic transition to a higher (lower) energy state, occurs simultaneosly with an annihilation (creation) of a photon as signaled by the negative (positive) sign of the complex exponential $\exp(-i\omega T)$ ($\exp(i\omega T)$)—the exponential stems from the positive (negative) part of the vector potential Eq. (1), which in a second-quantization formalism, will be accompanied by an annihilation (creation) photon operator.

Let us imagine a different situation from that illustrated in Fig. 1, that of an initial state $n_0<0$, i.e. lying to the left of the band minimum. The selection rule on the total angular momentum $\delta_{n-n_0,2}$ in the first term of Eq. (18) tells us that this is a transition to a state n such that $n>n_0$, to the right of n_0. Since this term is $H^{(+)}$, it corresponds to a photon annihilation; however, the transition leaves the electron in a lower energy state, giving rise to a *virtual* process. Virtual processes serve as intermediate states in a sequence of transitions ending in energy conservation, or they may happen in a time scale compatible with the Heisenberg's uncertainty principle. In general, the situation is somewhat complicated, and the type of transition depends on the values of n_0, σ and l. Using large values of the OAM can create a large difference between transitions, and one effectively may neglect some of them: non-resonant terms can be disregarded.

4.3. Induced polarization and current

Since both resonant and non-resonant terms ought to be considered, the current in intraband transitions is much smaller than expected for interband transitions. This is easily seen for the case of TL acting upon a QR having no SOI. When the SOI is not present, we can replace the wave functions by $\psi_m(\varphi)=e^{im\varphi}$—for a discussion on the difference between this and Eq. (4) see Ref. [7]. It is a simple matter to derive the matrix elements for the interaction with TL

$$\langle m'|H^{(+)}_{11}|m\rangle \quad = \quad \xi_\sigma e^{-i\omega t}\delta_{l+\sigma,m'-m}m\,. \tag{20}$$

To exemplify, let us take $l=1$ and $\sigma=1$, and study the evolution from an initial state $|m_0\rangle$. Then, the only non-vanishing terms are $\{a_{m_0},a_{m_0-2},a_{m_0+2}\}$. A calculation of the current density, after the light is turned off at $t=T$, yields, after integrating the current density in the whole ring

$$J \quad \simeq \quad \frac{q\hbar}{m}\left\{m_0+2|\xi_-|^2D^2m_0^3\right\}, \tag{21}$$

where D is a constant that depends on the sign of m_0, but for intraband transitions we expect $D(-|m_0|) \simeq D(|m_0|)$. The perturbation increases the rotation in the same direction the particle was originally moving, either right or left. Then, we conclude that there is no significant current for a balanced population of electrons in the QR.

5. Conclusions

We have studied the dynamics of electrons confined to semiconductor-based quantum rings under irradiation by twisted light, in the presence of Rashba spin-orbit coupling. We worked out the matrix elements of the light-matter interaction in the Coulomb gauge on the Rashba states of the quantum ring. Because the unperturbed states encode the SOI, the common light-matter interaction $\hat{\mathbf{p}} \cdot \mathbf{A}$, though not a spin-orbit coupling, can produce spin flips. We pointed out that the dynamics of orbital motion and spin exhibit two distinct time scales, and we have estimated these two in the case of small spin-orbit interaction. We point out that, for strong spin-orbit interaction as reported by Zhu *et al* [7], the spin dynamics can be faster than the orbital one. In view of the impossibility to apply the Rotating Wave Approximation, we studied the differences between optical excitation of inter- and intraband transitions. Thanks to the freedom to choose the value of OAM, we observe that resonant and non-resonant terms may be manipulated in order to gain control of specific transitions.

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