

Global analysis of the negative parity non-strange baryons in the $1/N_c$ expansion

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Abstract

A global study of the negative parity non-strange baryon observables is performed in the framework of the $1/N_c$ expansion. Masses, partial decay widths and photo-couplings are simultaneously analyzed. A main objective is to determine the composition of the spin $\frac{1}{2}$ and $\frac{3}{2}$ nucleon states, which come in pairs and involve two mixing angles which can be determined and tested for consistency by the mentioned observables. The issue of the assignment of those nucleon states to the broken $SU(4) \times O(3)$ mixed-symmetry multiplet is studied in detail, with the conclusion that the assignment made in the old studies based on the non-relativistic quark model is the preferred one. In addition, the analysis involves an update of the input data with respect to previous works.

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I. INTRODUCTION

The excited baryon states that correspond in the constituent quark model to the first radial and orbital excitations fit well into respectively a positive parity symmetric and a negative parity mixed-symmetric irreducible representation of the spin-flavor group $SU(2N_f)$, where N_f is the number of light flavors. For $N_f = 2$ both representations are 20-dimensional and are respectively denoted by **20** and **20'**. The negative parity non-strange excited baryons states are empirically the best known ones [1], and are organized in the $[20', \ell = 1]$ representation of $SU(4) \times O(3)$ [2]. Their masses, partial decay widths and photo-couplings are known to a degree where detailed analyses as the one presented here can be carried out and lead to some clear conclusions. The states consist of two spin 1/2 nucleon states: N(1535) and N(1650), two spin 3/2 nucleon states: N(1520) and N(1700), one spin 5/2 nucleon state: N(1675) and two isospin multiplets: $\Delta(1620)$ and $\Delta(1700)$ of spin 1/2 and 3/2 respectively. A very important issue, central to the study in this work, is the assignment of the physical states to the pairs of spin 1/2 and 3/2 nucleon states, and also the accurate determination of the corresponding mixing angles $\theta_{2J=1}$ and θ_3 . In order to settle that issue in as model independent as possible fashion it is necessary to carry out a simultaneous analysis of masses, partial decays widths and photo-couplings, as presented in this work. The framework used here is the $1/N_c$ expansion as presented elsewhere for masses [3, 4], decays [5] and photo-couplings [6].

The advantage of using the $1/N_c$ expansion [7] is that, in principle, the analysis is based on a framework which is true to QCD. It provides a justification for the approximate spin-flavor symmetry observed in baryons [8–11], which permits the implementation of the $1/N_c$ starting from that symmetry limit. From multiple applications it transpires that the $1/N_c$ expansion works well in the real world with $N_c = 3$ [18]. In particular, its implementation in the analysis of excited baryons (see e.g. Refs. [3–6, 12–17]), has shown to be phenomenologically consistent in the sense that the sub-leading corrections are of natural size. In the present work, the $1/N_c$ expansion is implemented as an operator expansion, where the coefficients are determined by the QCD dynamics, and those coefficients are evaluated by fitting to the pertinent observables. In addition, since usually at the first sub-leading order the number of observable quantities exceeds the number of operators, there are relations independent of the coefficients, which are exact up to sub-sub-leading order corrections, and which should

serve as a good test of the framework. The physics of excited baryons thus gets sorted out hierarchically in powers of $1/N_c$, helping in this way to define and organize the discussion of the different effects. It should be noted that the implementation for the negative parity baryons is carried out under the assumption that $SU(4) \times O(3)$ configuration mixings, i.e. mixings of different irreducible representations of that group, can be disregarded. A discussion of such effects was presented in [19]. In the $1/N_c$ expansion, baryon observables are represented by effective operators which are themselves decomposed in an operator basis which is organized according to the $1/N_c$ power counting of the operators. The coefficients of that decomposition are determined by the QCD dynamics. These are obtained by fitting the corresponding observables to experimental data.

In the case of the negative parity baryons, the analysis of the masses suffers from the four-fold ambiguity mentioned above in the assignment of the spin 1/2 and 3/2 nucleon physical states to the theoretical ones. It turns out that such ambiguity cannot be resolved by analyzing the masses alone, as acceptable fits, i.e. $\chi^2_{dof} \sim 1$ can be obtained for all four assignments. Because the strong decay partial widths with emission of a single pion, and the EM transitions are more sensitive to those assignments, they must be used to arrive at a definite conclusion. Historically, the first such analysis was carried out in the framework of the quark model [20]. In the $1/N_c$ expansion the work of Pirjol and Schat [21] was the first to address the problem, and it was also piecemeal addressed in other works on strong decays [5] and EM transitions [6]. However, an exhaustive combined analysis was still lacking, which is now presented in this work.

The present work is organized as follows. Sec. II provides a brief review of the framework used to analyze the non-strange low-lying negative parity excited baryons in the framework of the $1/N_c$ expansion of QCD. Some parameter independent relations valid to $\mathcal{O}(N_c^0)$ are also presented and discussed in this section. Sec. III describes the procedure followed to perform the global fits of the empirically known baryon properties to $\mathcal{O}(1/N_c)$ and analyze the corresponding results. The main conclusions are presented in Sec. IV. Finally, includes two Appendices: Appendix I gives the lists of operators furnishing the bases for the analysis of the baryon properties considered in this work, while Appendix II gives some parameter independent relations satisfied at LO by the photo-couplings.

II. PROPERTIES OF THE LOW-LYING NON-STRANGE NEGATIVE PARITY BARYONS IN THE $1/N_c$ EXPANSION

In the following a brief description of the $1/N_c$ expansion excited baryon framework is presented.

The group $SU(4)$ has 15 generators, namely, the spin, isospin and spin-isospin generators: S^i, T^a, G^{ia} , with $i = 1, 2, 3$ and $a = 1, 2, 3$, normalized to fulfill the following commutation relations:

$$\begin{aligned} [S^i, S^j] &= i\epsilon^{ijk}S^k, [T^a, T^b] = i\epsilon^{abc}T^c, [S^i, T^a] = 0, \\ [G^{ia}, G^{jb}] &= \frac{i}{4}\delta^{ij}\epsilon^{abc}T^c + \frac{i}{4}\delta^{ab}\epsilon^{ijk}S^k, \\ [S^i, G^{ja}] &= i\epsilon^{ijk}G^{ka}, [T^a, G^{ib}] = i\epsilon^{abc}G^{ic}. \end{aligned} \quad (1)$$

The matrix elements of G^{ia} between baryons with spin of order N_c^0 are $\mathcal{O}(N_c)$.

Introducing the spherical notation for the group generators and omitting the projections for simplicity, the generators are denoted by:

$$S^{[1,0]}, \quad T^{[0,1]} \quad \text{and} \quad G^{[1,1]}, \quad (2)$$

where the notation $\Lambda^{[J,I]}$ implies that the operator Λ transforms as the $[J, I]$ irreducible representation of the $SU(2) \times SU(2)$ spin \times isospin subgroup of $SU(4)$.

A. $[20', 1^-]$ states for arbitrary N_c

For arbitrary N_c , the states of interest here belong to the $SU(4)$ mixed-symmetric representation $(p, q, r) = (N_c - 2, 1, 0)$, which has dimension $((N_c - 1)(N_c + 2)!)/2N_c!$. These states are constructed by coupling a fundamental $SU(4)$ state to a totally symmetric state with $N_c - 1$ indices, which is denoted as the *core* [4]:

$$\overbrace{\square \square \dots \square}^{N_c-1} \otimes \square = \overbrace{\square \square \dots \square}^{N_c} \oplus \overbrace{\square \square \dots \square}^{N_c-1} \text{ MS}, \quad (3)$$

Denoting the core states by $|S_c, S_{c3}, I_{c3}\rangle$, where these states belong to the symmetric $SU(4)$ representation in which $I_c = S_c$ is satisfied, the mixed-symmetric states of interest denoted by $|S, S_3; I, I_3\rangle$ are given by [4]:

$$|S, S_3; I, I_3\rangle_{\mathbf{MS}} = \sum_{\xi=\pm\frac{1}{2}} C_{\mathbf{MS}}(I, S, \xi) |S, S_3; I, I_3; S_c = S + \xi\rangle, \quad (4)$$

where

$$C_{\mathbf{MS}}(I, S, \pm\frac{1}{2}) = \begin{cases} 1 & \text{si } I = S \pm 1 \\ 0 & \text{si } I = S \mp 1 \\ \pm \sqrt{\frac{(2S+1\pm 1)(N_c+1\pm(2S+1))}{2N_c(2S+1)}} & \text{si } I = S \end{cases}, \quad (5)$$

and

$$\begin{aligned} |S, S_3; I, I_3; S_c\rangle &= \sum_{s_3, i_3} \langle S_c, S_3 - s_3; 1/2, s_3 | S, S_3 \rangle \langle I_c = S_c, I_3 - i_3; 1/2, i_3 | I, I_3 \rangle \\ &\times |S_c, S_3 - s_3; I_c = S_c, I_3 - i_3\rangle \otimes |1/2, s_3; 1/2, i_3\rangle. \end{aligned} \quad (6)$$

S is the total spin of the baryon associated with the spin group $SU(2)$, S_c is the core's spin. The $SU(2)$ Clebsch-Gordan coefficients are defined with the standard Condon-Shortley phase convention (see e.g. Ref. [22]).

The states belonging to the $[20', 1^-]$ of $SU(4) \times O(3)$ are now expressed by including the orbital angular momentum,

$$|J, J_3; I, I_3; \ell, S\rangle_{\mathbf{MS}} = \sum_m \langle \ell, m; S, J_3 - m | J, J_3 \rangle | \ell, m \rangle \otimes |S, J_3 - m; I, I_3\rangle_{\mathbf{MS}}. \quad (7)$$

The symmetric states are given explicitly in Ref. [5].

The state mixings mentioned in the Introduction are induced by interactions that break spin/isospin symmetry. These mixings are between the pairs of states that are in the two octets with the same $J = \frac{1}{2}$ or $\frac{3}{2}$. The mixings are defined according to:

$$\begin{pmatrix} N_J \\ N'_J \end{pmatrix} = \begin{pmatrix} \cos \theta_{2J} & \sin \theta_{2J} \\ -\sin \theta_{2J} & \cos \theta_{2J} \end{pmatrix} \begin{pmatrix} {}^2N_J^* \\ {}^4N_J^* \end{pmatrix}, \quad (8)$$

where $N_J^{(*)}$ are mass eigenstates. The mixing angles are determined by the diagonalization of the corresponding 2×2 mass matrices. The eigenvalues and mixing angles are given by:

$$m_{N_J, N'_J} = \frac{M_2^J + M_4^J}{2} \pm \text{sign}(M_2^J - M_4^J) \sqrt{\left(\frac{M_2^J - M_4^J}{2}\right)^2 + (M_{24}^J)^2}, \quad (9)$$

$$\theta_{2J} = \frac{1}{2} \arctan \left(\frac{2M_{24}^J}{M_2^J - M_4^J} \right) \pmod{\pi}, \quad (10)$$

where $M_2^J = \langle {}^2N_J^* | \mathbf{M} | {}^2N_J^* \rangle$, etc. Angles can be defined to be in the interval $[0, \pi)$ by conveniently choosing the phase of the physical states.

As already mentioned there are four possible assignments of the states N_J and N_J' to the physical ones. They are:

$$\begin{aligned} \text{Set 1} &= \left\{ N_{\frac{1}{2}} = N(1535), N_{\frac{1}{2}}' = N(1650), N_{\frac{3}{2}} = N(1520), N_{\frac{3}{2}}' = N(1700) \right\} \\ \text{Set 2} &= \left\{ N_{\frac{1}{2}} = N(1650), N_{\frac{1}{2}}' = N(1535), N_{\frac{3}{2}} = N(1520), N_{\frac{3}{2}}' = N(1700) \right\} \\ \text{Set 3} &= \left\{ N_{\frac{1}{2}} = N(1535), N_{\frac{1}{2}}' = N(1650), N_{\frac{3}{2}} = N(1700), N_{\frac{3}{2}}' = N(1520) \right\} \\ \text{Set 4} &= \left\{ N_{\frac{1}{2}} = N(1650), N_{\frac{1}{2}}' = N(1535), N_{\frac{3}{2}} = N(1700), N_{\frac{3}{2}}' = N(1520) \right\} \end{aligned} \quad (11)$$

It should be noted that in previous works [3, 4, 21] the mixing angles have been defined using some specific state assignment. For example, in Ref. [21] they are defined in their Eqs. (18) and (19), which only correspond to the definition Eq. (10) when states are assigned according to Set 1 in Eq. (11).

B. Operator analysis

A local QCD color singlet operator, expressed in terms of the quark and gluon fields and which has irreducible transformation properties under isospin and rotations, can be expanded in the subspace of baryon states in powers of $1/N_c$ as an operator expansion consisting of composite operators built with products of operators which are tensors of $SU(4) \times O(3)$ [23, 24]. Those tensor operators have a well defined $1/N_c$ power counting, as described below. Thus, a QCD operator has the $1/N_c$ expansion of the form:

$$\mathcal{O}_{QCD} = \sum_{n,\ell} C_\ell^{(n)} \mathcal{O}_\ell^{(n)}. \quad (12)$$

The $1/N_c$ order of the composite operators is determined by the n -body character or degree of these, which is given by the number of quark fields necessary to built an operator with the same transformation properties [25], and by the N_c order of the matrix elements of the tensor operators appearing as factors. Since an n -body operator requires the exchange of $n - 1$ gluons to be produced, there is an overall factor $1/N_c^{n-1}$, and thus the (naive) order in $1/N_c$ of an n -body composite operator becomes:

$$\nu(\mathcal{O}^{(n)}) = n - 1 - \kappa, \quad (13)$$

where κ is given by the order of the factors, and n satisfies $n \leq N_c$. The coefficients $C_\ell^{(n)}$ in Eq. (12) are the unknown dynamical coefficients of $\mathcal{O}(N_c^0)$, which are determined by the QCD dynamics.

In the subspace of the **MS** states of interest, a basis of composite operators can be constructed by distinguishing two sets of generators according to whether they act on the core or the “excited quark” which corresponds to the spin-flavor index associated to the second row of the Young tableaux. Generators acting on the core will be denoted by $\{S_c^{[1,0]}, T_c^{[0,1]}, G_c^{[1,1]}\}$ and those acting on the excited quark by $\{\ell^{[1,0]}, s^{[1,0]}, t^{[0,1]}, g^{[1,1]}\}$. Then, n -body operators can be those consisting of the product of n $SU(4)$ generators acting only on the core, or $n-1$ $SU(4)$ generators acting on the core and at least one generator of $SU(4) \times O(3)$ acting on the excited quark. It should be noted that the order κ appearing in Eq. (13) takes the value $\kappa = 0$ for all the generators except the generator G_c for which $\kappa = 1$.

C. Mass operators

The basis of mass operators up to order $1/N_c$ is taken from that obtained in [4], and shown here in Table VII of Appendix I. Convenient normalization factors have been included in the definition of the basis operators in such a way that they have natural size matrix elements. There is one operator $\mathcal{O}(N_c)$, namely the identity or baryon number. There are only two operators $\mathcal{O}(N_c^0)$, both involving the coupling of the angular momentum ℓ . The first corresponds in the quark model picture to the Thomas precession spin-orbit interaction, which for baryons in **S** representation is $\mathcal{O}(1/N_c)$ but in the **MS** representation is $\mathcal{O}(N_c^0)$, and the second operator involves the couplings of the axial currents of the core and the excited quark through a second rank tensor $\ell^{(2)}$. There are four operators at $\mathcal{O}(1/N_c)$: M_4 is a linear combination of two $\mathcal{O}(N_c^0)$ operators, M_5 is a 2-body spin-orbit interaction involving the spin of the core, M_6 is the hyperfine core interaction which is known to play the main role in the spin-flavor symmetry breaking of the masses, M_7 is the hyperfine between core and excited quark which in states with $\ell = 1$ is expected to give suppressed matrix elements, and finally M_8 is the version of M_3 without flavor. The matrix elements of the basis operators were given (in a different normalization) in Table II of Ref. [4].

For the content of states for $N_c = 3$ there are seven possible masses and two mixing angles, which means at most nine independent mass operators. Up to $\mathcal{O}(N_c^0)$ there are only three

operators, and thus there are six parameter independent relations. As noted in Ref. [21], two of these relations are the values of the mixing angles, which are $\theta_1 = \cos^{-1}(-\sqrt{2/3}) = 2.526$ and $\theta_3 = \cos^{-1}(-\sqrt{5/6}) = 2.721$ in the present convention. It should be noted that using the mass formulas up to, and including $\mathcal{O}(N_c^0)$ corrections, one can identify with the nucleon states in the paper of Pirjol and Schat [21] in the following way: $N_{\frac{1}{2}} = N_{\frac{1}{2}}^{T=1}$, $N'_{\frac{1}{2}} = N_{\frac{1}{2}}^{T=0}$, $N_{\frac{3}{2}} = N_{\frac{3}{2}}^{T=2}$, $N'_{\frac{3}{2}} = N_{\frac{3}{2}}^{T=1}$, where T indicates the tower to which the state belongs as explained in [21]. Thus, their four alternative results for the $\mathcal{O}(N_c^0)$ mixing angles (see their Fig. 3) follow from the four possible assignments Eq. (11) which basically imply shifting each of the angles by $\pi/2$.

The remaining four relations involve only the physical masses and are as follows:

$$\begin{aligned}
M_{N_{\frac{1}{2}}} &= M_{N'_{\frac{3}{2}}} \\
M_{N_{\frac{3}{2}}} &= M_{N_{\frac{5}{2}}} \\
M_{N_{\frac{5}{2}}} - \frac{1}{10}M_{N'_{\frac{1}{2}}} &= 3 \left(\frac{1}{2}M_{\Delta_{\frac{1}{2}}} - \frac{1}{5}M_{\Delta_{\frac{3}{2}}} \right) \\
M_{\Delta_{\frac{1}{2}}} &= \frac{1}{6}M_{N_{\frac{1}{2}}} + \frac{5}{6}M_{N_{\frac{5}{2}}}
\end{aligned} \tag{14}$$

Numerically, these relations are satisfied best with Set 3 in agreement with the result of Ref. [21] where it was denoted “Assignment No.1”. However, as observed later at $\mathcal{O}(1/N_c)$ and after performing the combined fits with decays and EM amplitudes, a different assignment will be favored. The hyperfine interaction plays the key role in modifying the LO values of the mixing angles and in permitting a different identification of states.

Up to $\mathcal{O}(1/N_c)$ there are eight operators, which means that there will be one mass parameter independent relation which can be checked. Expressed in terms of the physical masses and mixing angles this relation reads

$$\begin{aligned}
&3 \left(M_{N_{\frac{1}{2}}} + M_{N'_{\frac{1}{2}}} - 4M_{N_{\frac{3}{2}}} - 4M_{N'_{\frac{3}{2}}} + 6M_{N_{\frac{5}{2}}} + 8M_{\Delta_{\frac{1}{2}}} - 8M_{\Delta_{\frac{3}{2}}} \right) = \\
&\left[13 \cos 2\theta_1 + \sqrt{32} \sin 2\theta_1 \right] \left(M_{N'_{\frac{1}{2}}} - M_{N_{\frac{1}{2}}} \right) - 4 \left[\cos 2\theta_3 - \sqrt{20} \sin 2\theta_3 \right] \left(M_{N'_{\frac{3}{2}}} - M_{N_{\frac{3}{2}}} \right).
\end{aligned} \tag{15}$$

Fig. 1 shows the correlation between the angles if the assignment of states is the one of Set 1 in Eq. (11). For the other possible assignments one just shifts the angles by $\frac{\pi}{2}$ correspondingly.

D. Strong decays

The partial decay widths for the strong decays of the negative parity excited baryons via the emission of a π or η meson are given by [5]:

$$\Gamma^{[\ell_P, I_P]} = \frac{k_P^{1+2\ell_P}}{8\pi^2 \Lambda^{2\ell_P}} \frac{M_{B^*}}{M_B} \left| \frac{\sum_q C_q^{[\ell_P, I_P]} \mathcal{B}_q(\ell_P, I_P, S, I, J^*, I^*, S^*)}{\sqrt{(2J^* + 1)(2I^* + 1)}} \right|^2, \quad (16)$$

where the asterisk refers to the excited baryon, $\ell_P = 0$ or 2 is the angular momentum of the meson and I_P its isospin, and Λ is an arbitrary scale introduced for convenience so that all quantities in the sum are dimensionless. By choice in the calculations, $\Lambda = 200$ MeV will be taken. $\mathcal{B}_q(\ell_P, I_P, S, I, J^*, I^*, S^*)$ are reduced matrix elements defined via the Wigner-Eckart theorem as follows:

$$\begin{aligned} \mathbf{S} \langle S, S_3; I, I_3 \mid \left(B_{[m_P, I_{P3}]}^{[\ell_P, I_P]} \right)_q \mid J^*, J_3^*; I^*, I_3^*; S^* \rangle \mathbf{MS} &= \frac{(-1)^{\ell_P - J^* + S + I_P - I^* + I}}{\sqrt{(2S + 1)(2I + 1)}} \times \\ \langle \ell_P, m_P; J^*, J_3^* \mid S, S_3 \rangle \langle I_P, I_{P3}; I^*, I_3^* \mid I = S, I_3 \rangle \mathcal{B}_q(\ell_P, I_P, S, I, J^*, I^*, S^*) . \end{aligned} \quad (17)$$

The formulas above hold if the physical states have well defined quark-spin S^* . However, the physical states are a mix of the $S^* = J^* \pm \frac{1}{2}$ quark spin states, and thus they need to be modified in the obvious way via the corresponding mixing of the reduced matrix elements. This of course only happens for the case of the $N_{\frac{1}{2}}$ and $N_{\frac{3}{2}}$ states, and thus the dependence of these partial widths on the angles θ_1 and θ_3 . Note that the analysis is restricted to the S and D-wave decays, because the only possible G-wave transition in the decay $N_{\frac{5}{2}} \rightarrow \Delta\pi$ is not yet empirically sorted out, and is also irrelevant to the main discussion of this work. The corresponding bases of baryon decay operators and their associated matrix elements were obtained in Ref. [5]. For convenience those operator bases are depicted in Table VIII of Appendix I. All $\mathcal{O}(N_c^0)$ operators are 1-body type except for one of the 2-body D-wave for π emission. For S-wave π emission there is one $\mathcal{O}(N_c^0)$ and three $\mathcal{O}(1/N_c)$. For D-wave π emission there two $\mathcal{O}(N_c^0)$ and six $\mathcal{O}(1/N_c)$ (two of which are 3-body). For S-wave η emission there is one $\mathcal{O}(N_c^0)$ and one $\mathcal{O}(1/N_c)$.

E. Photoproduction helicity amplitudes

The multipole components of the helicity amplitudes for photo-production of the $[20', 1^-]$ states can be expressed in terms of the matrix elements of effective operators as follows [6]:

$$\begin{aligned}
A_{\lambda}^{ML} &= \sqrt{\frac{3\alpha N_c}{4\omega}} (-1)^{L+1} \eta(B^*) \sum_{n,I} g_{n,L}^{[L,I]}(\omega) \left\langle J^*, \lambda; I^*, I_3; S^* \mid (\mathcal{B}_n)_{[1,0]}^{[L,I]} \mid \frac{1}{2}, \lambda-1; \frac{1}{2}, I_3 \right\rangle, \\
A_{\lambda}^{EL} &= \sqrt{\frac{3\alpha N_c}{4\omega}} (-1)^L \eta(B^*) \sum_{n,I} \left[\sqrt{\frac{L+1}{2L+1}} g_{n,L-1}^{[L,I]}(\omega) + \sqrt{\frac{L}{2L+1}} g_{n,L+1}^{[L,I]}(\omega) \right] \\
&\quad \times \left\langle J^*, \lambda; I^*, I_3; S^* \mid (\mathcal{B}_n)_{[1,0]}^{[L,I]} \mid \frac{1}{2}, \lambda-1; \frac{1}{2}, I_3 \right\rangle, \tag{18}
\end{aligned}$$

where ML and EL indicates the respective multipoles. Note that, due to parity conservation, only $E1$, $M2$ and $E3$ multipoles are allowed. Moreover, $\lambda = \frac{1}{2}, \frac{3}{2}$ is the helicity defined along the \hat{z} -axis which coincides with the photon momentum, and $\omega = (M_{B^*}^2 - M_N^2)/2M_{B^*}$ is the photon energy in the rest frame of the excited baryon B^* . These amplitudes correspond to the standard definition as used by the Particle Data Group [1], which includes a sign factor $\eta(B^*)$ that stems from the strong decay amplitude of the excited baryon to a πN state making them independent of the phase conventions used to define the excited states. The sign factors

$$\eta(B^*) = (-1)^{J^* - \frac{1}{2}} \text{sign}(\langle \ell_{\pi} N \mid H_{QCD} \mid J^* I^* \rangle) \tag{19}$$

are on the other hand convention dependent. The sum over n is over all available basis operators with the given $[L, I]$ quantum numbers. The factor $\sqrt{N_c}$ appears as usual for transition matrix elements between excited and ground state baryons [19]. In the electric multipoles there is a combination of the coefficients $g_{n,L-1}^{[L,I]}$ and $g_{n,L+1}^{[L,I]}$, and because the operators appearing in these multipoles do not appear in the magnetic multipoles, one may as well replace that combination of coefficients by a single term without any loss of generality. Thus, in what follows only $g_{n,L-1}^{[L,I]}$ is kept. These and the coefficients $g_{n,L}^{[L,I]}$ are going to be determined by fits to the empirical helicity amplitudes. In order to streamline the notation the coefficients will then be denoted by: $XL_n^{(I)}$, $X = E$ or M , and $I = 0, 1$.

As in the case of decays, one must take into account the effects of state mixing for the physical amplitudes.

The corresponding bases of operators and their associated matrix elements were obtained in Ref. [6]. For convenience the list is depicted in Table IX of Appendix I.

As in the case of the masses, which are linear in the effective couplings, there are linear relations for the photo-couplings which are independent of those effective couplings. Those relations at LO which involve either only neutral or only charged baryons are displayed in the Appendix II. Their tests are at this point not significant because the errors of the empirical amplitudes are still too large.

III. GLOBAL ANALYSIS AND DISCUSSION OF RESULTS

As discussed in the Introduction, the main aim of the present work is to perform a global and consistent fit of the masses, strong decay widths and EM helicity amplitudes associated with the low-lying negative parity baryons. A summary of the present empirical knowledge of their Breit-Wigner masses, strong decay partial widths and EM helicity amplitudes for photo-production taken from Ref. [1] is given in Table I.

In order to perform the fits one proceeds as follows. Starting with a certain set of initial values for the parameters associated to the mass operators one determines the theoretical masses and mixing angles. Next, with these values of the mixing angles and a set of values for the strong decay parameters, one obtains the theoretical predictions for the strong widths and the strong amplitude signs needed for the calculation of the EM helicity amplitudes, Eq. (19). Finally, using again the same mixing angles together with the strong signs just determined and a set of values for the helicity amplitudes parameters, one obtains the theoretical predictions for the EM helicity amplitudes. With all these theoretical predictions f_i^{theo} , and the existing empirical values f_i^{emp} and associated errors summarized in Table I, the $\chi_{dof}^2 = \sum_i [(f_i^{theo} - f_i^{emp}) / \Delta f_i^{emp}]^2 / n_{dof}$ is calculated. Here, $i = 1, \dots, N_{emp}$ runs over the different empirically known observables and $n_{dof} = N_{emp} - N_{ip}$, where N_{ip} is the total number of independent parameters used in the fit. This procedure is iterated using different sets of values for mass, strong decay and EM amplitudes parameters until a (global) minimum of χ_{dof}^2 is found. The minimization is carried out using the MINUIT program.

Using the procedure mentioned above, fits for the four possible assignments Eq. (11) were performed. It is found that in all cases it is possible to obtain values of $\chi_{dof}^2 < 1$. However, these fits have qualitatively very different characteristics. This can be observed in Figs. 2 and 3, which depict the coefficients associated with mass and strong decay operators and with the EM amplitude operators, respectively.

In order to decide which assignments are acceptable from a physical point of view one needs to consider a certain number of criteria. The first one has to do with the naturalness of the coefficients. In fact, if the $1/N_c$ expansion is assumed to be an appropriate framework, all the coefficients should be within a natural range in magnitude. It should be noted that in the case of the strong decays and helicity amplitudes this criterion should be applied to each partial wave separately. A second criterion is related to the assumption that in the strong and EM amplitudes, when 1-body operators can contribute they should not be suppressed with respect to 2-body operators. This assumption is based on the fact that the quark model description of these amplitudes, where it is assumed by construction the dominance 1-body operators, is quite successful. Finally, the third criterion takes into account the importance of the hyperfine interaction in the description of the masses. In fact, this interaction plays a crucial role in explaining the lightness of the singlet $\Lambda(1405)$ and $\Lambda(1520)$, whose masses are pushed down by the hyperfine interaction as shown in [14], and as realized in the early paper by Isgur [26]. The lightness of those Λ baryons is also observed in recent lattice QCD calculation of the baryon spectrum [28], where due to the larger quark masses and the possible suppression of the finite width effects due to the implementation of the lattice calculation, the hyperfine interaction is clearly driving the downward shift of the masses. The result of the application of these criteria to the results shown in Figs. 2 and 3 is summarized in Table II. From that table one can conclude that the preferred assignment of states is that of Set 1. This assignment is the one that had been adopted in the early works with the quark model [20, 26, 27] and in the various analysis in the $1/N_c$ expansion, [3–6, 12, 21]. The detailed analysis presented here further establishes that scheme.

The numerical results for the operator coefficients corresponding to the favored Set 1 are given in Table IV. Two possibilities have been considered: Fit 1 includes all the independent operators appearing in the basis of each observable, while in Fit 2 those operators whose coefficients are compatible with zero have been excluded. More specifically, Fit 2 was obtained by taking the result of Fit 1 and removing those operators whose coefficients are compatible with zero in a subsequent way. In the case that at certain step there were more than one operator in this situation, those with higher relative error were removed.

It is interesting to compare the operator coefficients obtained in the present global analysis with those reported in Refs. [4–6] where fits were made independently for each baryon property. In making this comparison one should keep in mind that some new empirical val-

ues became available since those analysis were performed. In the case of strong decays it is observed that except for a few exceptions the coefficients obtained in global and non-global analysis are quite similar when error bars are taken into account. One of the exceptions concerns the coefficients $C_i^{[0,1]}$ with $i = 2, 3, 4$ which showed a degeneracy in the non-global fit. Such degeneracy is removed in the present global analysis. The other is related to the coefficients $C_{7,8}^{[2,1]}$ associated to 3-body operators which were not taken into account in Ref. [5]. While the present analysis indicates that $C_7^{[2,1]}$ is indeed compatible with zero, $C_8^{[2,1]}$ turns out to be comparable to the coefficient of the corresponding 1-body operator. On the other hand, note that $C_2^{[2,1]}$ which was non-vanishing in the previous analysis turns to be compatible with zero here. Finally, $C_2^{[0,0]}$ which was not included in non-global analysis is also compatible with zero here. Making a comparison between the present and previous works on EM decays [6], one notices that the values of the coefficient do not change appreciably. In most cases the average values are very similar, and they overlap within errors. It is interesting to revisit the Moorhouse rule [29], which states that the EM transitions of proton excited states with quark spin $S^* = \frac{3}{2}$ are suppressed. An explanation of the rule in the quark model stems from coupling the photon to the constituent quarks with the isoscalar/isovector strength determined by the quark charges. State mixing and the presence of 2-body operators give violations to the rule. The Table VI gives the quark-spin content of the proton states as determined from the mixing angles obtained in the fits to Set 1 and a check if the rule is closely satisfied. This check represents a good test of the identification of states corresponding to the Set 1. For Sets 3 and 4 the only way to make the suppressions work is through extra strength in the 2-body operators as shown in Fig. 3, which is expected to be unnatural. The case of $p(1675)$ is a particularly unambiguous test as it is unaffected by mixings [29].

In the strong decays the following selection rules result at LO, i.e., when the mixing angles are the ones obtained in the expansion of the masses up to $\mathcal{O}(N_c^0)$ and at LO in the decay amplitudes (1-body operators): for the S-wave decays $N_{\frac{1}{2}} \rightarrow \eta N$, $N'_{\frac{1}{2}} \rightarrow \pi N$, $N_{\frac{3}{2}} \rightarrow \pi \Delta$, and for the D-wave decay $N'_{\frac{1}{2}} \rightarrow \pi \Delta$. These can be checked using the Tables III, IV and V of Ref. [5]. Since the mixing angle θ_1 changes by more than $\pi/2$ from LO to NLO, one expects that for the $J = 1/2$ nucleons the selection rules will be poorly satisfied. In fact the decays $N_{\frac{1}{2}} \rightarrow \eta N$, $N'_{\frac{1}{2}} \rightarrow \pi N$ are not suppressed at all, while the D-wave decay $N'_{\frac{1}{2}} \rightarrow \pi \Delta$ is quite suppressed with respect to the S-wave one, $N'_{\frac{1}{2}} \rightarrow \pi N$, but this is simply

because of being a higher partial wave. The large change in the angle θ_1 from LO to NLO is due to the fact that the mass operators of $\mathcal{O}(N_c^0)$ are quite weak and the NLO hyperfine mass operator is very strong, leading to a large rearrangement in the composition of the $J = 1/2$ nucleon states. Finally the suppression of the S-wave decay $N_{\frac{3}{2}} \rightarrow \pi\Delta$ should be well fulfilled because the angle θ_3 does not change too much from LO to NLO. This seems to be indeed the case as the D-wave decay $N_{\frac{3}{2}} \rightarrow \pi\Delta$ has similar rate to that S-wave decay.

IV. CONCLUSIONS

The determination of the spin-flavor structure of the negative parity baryons is of fundamental importance in the study of excited baryons, as it serves to unravel the QCD dynamics responsible for their properties. Of particular importance is the structure of the $J = 1/2$ and $3/2$ nucleons, which coming in pairs can mix. The mixing is a sensitive measure of that dynamics. The connection between the theoretical states and the observed ones is a priori a four-fold undetermined problem. The physics is definitely different in each case, as it has been shown in this work through the global analysis of observables. The study comes to the conclusion that there is a favored assignment, based on the three criteria outlined in section III. Set 1 is acceptable on all counts, while Set 2 comes second, failing only in one criterion, namely that 2-body operators dominate over 1-body ones in the strong decays. Set 1 is moreover the favored one in the simplest versions of the quark model with suppressed spin-orbit interaction, which indicates that the seemingly simplest dynamics is the most realistic one. Concerning the mixing angles, this work finds $(\theta_1, \theta_3) = (0.49 \pm 0.29, 3.01 \pm 0.17)$ and $(0.40 \pm 0.13, 2.96 \pm 0.05)$ for Fits 1 and 2 respectively. For comparison in a previous non-global analysis of the strong decays using the $1/N_c$ [5], the value $\theta_1 = 0.39 \pm 0.11$ was obtained and θ_3 exhibited a two-fold degeneracy $\theta_3 = (2.38, 2.82) \pm 0.11$. In this sense the present global analysis is important to remove such degeneracy in the values for θ_3 . It is useful to compare the results obtained in the present analysis with old results obtained with analyses based on the quark model and of the $SU(6)_W$ approach. The present values are in good agreement with the earlier quark model determination $(\theta_1, \theta_3) = (0.55, 3.03)$, obtained from the analysis of strong decays alone [27]. The present results for θ_3 agree with old results in the $SU(6)_W$ approach [30], while for θ_1 those references disagreed with each other, for the most part because the input partial widths for the $N_{\frac{1}{2}}$ baryons had changed in

the meantime, with the latest agreeing with the present result. Finally, it is worthwhile to stress that further improvement in the empirical inputs for partial decay widths and helicity amplitudes is necessary in order to reduce the mixing angles uncertainties and to establish with an even stronger emphasis Set 1 as the correct assignment of spin-flavor states.

ACKNOWLEDGEMENTS

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APPENDIX I: BASES OF OPERATORS

For the reader's convenience the operator bases are shown: Table VII gives the mass operators, Table VIII operators for the strong decays and Table IX operators for the helicity amplitudes. They were respectively obtained in Refs. [4–6], where the corresponding matrix elements relevant for the present work can be also found.

APPENDIX II: PARAMETER INDEPENDENT RELATIONS FOR HELICITY AMPLITUDES

The parameter independent relations satisfied at LO by the photo-couplings are shown below. At LO the relations are given separately for the neutral and unit charge baryons. Other LO relations involving simultaneously neutral and charged baryons are not given here, but can be easily obtained. In the following the notation $s_i \equiv \sin \theta_i$, $c_i \equiv \cos \theta_i$ is used.

Charged baryon relations:

$$\begin{aligned}
& \left(\sqrt{\frac{2}{3}} c_1 c_3 + 2\sqrt{\frac{6}{5}} c_1 s_3 + \frac{2}{\sqrt{15}} s_1 s_3 \right) A_{\frac{1}{2}}^p[N_{\frac{1}{2}}] + \frac{1}{\sqrt{3}} A_{\frac{1}{2}}^p[N_{\frac{3}{2}}] + A_{\frac{3}{2}}^p[N_{\frac{3}{2}}] \\
& + \left(\frac{2}{\sqrt{15}} c_1 s_3 - \sqrt{\frac{6}{5}} s_1 s_3 - \sqrt{\frac{2}{3}} s_1 c_3 \right) A_{\frac{1}{2}}^p[N'_{\frac{1}{2}}] + \left(\frac{2}{9} (\sqrt{10} + 2\sqrt{15}) s_3 - \frac{1}{\sqrt{3}} c_3 \right) A_{\frac{1}{2}}^N[\Delta_{\frac{1}{2}}] \\
& - \left(\frac{1}{\sqrt{6}} c_3 + \frac{10+19\sqrt{6}}{18\sqrt{5}} s_3 \right) A_{\frac{1}{2}}^N[\Delta_{\frac{3}{2}}] - \left(\frac{1}{\sqrt{2}} c_3 + \frac{57+5\sqrt{6}}{9\sqrt{10}} s_3 \right) A_{\frac{3}{2}}^N[\Delta_{\frac{3}{2}}] = 0 \\
\\
& \left(2\sqrt{\frac{6}{5}} c_1 c_3 + \frac{2}{\sqrt{15}} s_1 c_3 - \sqrt{\frac{2}{3}} c_1 s_3 \right) A_{\frac{1}{2}}^p[N_{\frac{1}{2}}] + \left(\frac{2}{\sqrt{15}} c_1 c_3 - 2\sqrt{\frac{6}{5}} s_1 c_3 + \sqrt{\frac{2}{3}} c_1 s_3 \right) A_{\frac{1}{2}}^p[N'_{\frac{1}{2}}] \\
& + \frac{1}{\sqrt{3}} A_{\frac{1}{2}}^p[N_{\frac{3}{2}}] + A_{\frac{3}{2}}^p[N_{\frac{3}{2}}] + \left(\frac{2}{9} (\sqrt{10} + 2\sqrt{15}) c_3 + \frac{1}{\sqrt{3}} s_3 \right) A_{\frac{1}{2}}^N[\Delta_{\frac{1}{2}}] \\
& + \left(\frac{1}{\sqrt{6}} s_3 - \frac{10+19\sqrt{6}}{18\sqrt{5}} c_3 \right) A_{\frac{1}{2}}^N[\Delta_{\frac{3}{2}}] + \left(\frac{1}{\sqrt{2}} s_3 - \frac{57\sqrt{10}+10\sqrt{15}}{90} c_3 \right) A_{\frac{3}{2}}^N[\Delta_{\frac{3}{2}}] = 0 \\
\\
& \frac{1}{3\sqrt{5}} \left(8 c_1 + \frac{1}{\sqrt{2}} s_1 \right) A_{\frac{1}{2}}^p[N_{\frac{1}{2}}] + \frac{\sqrt{2}}{3} (2\sqrt{5} c_3 + s_3) A_{\frac{1}{2}}^p[\Delta_{\frac{1}{2}}] + \frac{1}{\sqrt{2}} A_{\frac{1}{2}}^p[N_{\frac{5}{2}}] + A_{\frac{3}{2}}^p[N_{\frac{5}{2}}] \\
& + \frac{1}{3\sqrt{5}} \left(\frac{1}{\sqrt{2}} c_1 - 8 s_1 \right) A_{\frac{1}{2}}^{p'}[N_{\frac{1}{2}}] + \frac{\sqrt{2}}{3} (c_3 - 2\sqrt{5} s_3) A_{\frac{1}{2}}^{p'}[\Delta_{\frac{1}{2}}] \\
& + \frac{2\sqrt{15}-3\sqrt{10}}{54} A_{\frac{1}{2}}^N[\Delta_{\frac{1}{2}}] + \frac{663-5\sqrt{6}}{108\sqrt{5}} A_{\frac{1}{2}}^N[\Delta_{\frac{3}{2}}] - \frac{5\sqrt{10}+139\sqrt{15}}{180} A_{\frac{3}{2}}^N[\Delta_{\frac{3}{2}}] = 0
\end{aligned} \tag{20}$$

Neutral baryon relations:

$$\begin{aligned}
& \left(\sqrt{\frac{2}{3}} c_1 c_3 + 2\sqrt{\frac{6}{5}} c_1 s_3 + \frac{2}{\sqrt{15}} s_1 s_3 \right) A_{1/2}^n[N_{1/2}] + \frac{1}{\sqrt{3}} A_{1/2}^n[N_{3/2}] + A_{1/2}^n[N_{3/2}] \\
& + \left(\frac{2}{\sqrt{15}} c_1 s_3 - \sqrt{\frac{2}{3}} s_1 c_3 - 2\sqrt{\frac{6}{5}} s_1 s_3 \right) A_{1/2}^n[N'_{1/2}] + \left(\frac{1}{\sqrt{3}} c_3 - \frac{2\sqrt{5}}{9} (\sqrt{2} + 2\sqrt{3}) s_3 \right) A_{1/2}^N[\Delta_{1/2}] \\
& + \left(\frac{1}{\sqrt{6}} c_3 + \frac{\sqrt{5}}{90} (10 + 19\sqrt{6}) s_3 \right) A_{1/2}^N[\Delta_{3/2}] + \left(\frac{1}{\sqrt{2}} c_3 + \frac{\sqrt{5}}{90} (57\sqrt{2} + 10\sqrt{3}) s_3 \right) A_{3/2}^N[\Delta_{3/2}] = 0 \\
\\
& \left(2\sqrt{\frac{6}{5}} c_1 c_3 + \frac{2}{\sqrt{15}} s_1 c_3 - \sqrt{\frac{2}{3}} c_1 s_3 \right) A_{1/2}^n[N_{1/2}] + \left(\frac{2}{\sqrt{15}} c_1 c_3 - 2\sqrt{\frac{6}{5}} s_1 c_3 + \sqrt{\frac{2}{3}} s_1 s_3 \right) A_{1/2}^n[N'_{1/2}] \\
& + \frac{1}{\sqrt{3}} A_{1/2}^n[N'_{3/2}] + A_{3/2}^n[N'_{3/2}] - \left(\frac{2}{9} (\sqrt{10} + 2\sqrt{15}) c_3 + \frac{1}{\sqrt{3}} s_3 \right) A_{1/2}^N[\Delta_{1/2}] \\
& + \left(\frac{10\sqrt{5}+19\sqrt{30}}{90} c_3 - \frac{1}{\sqrt{6}} s_3 \right) A_{1/2}^N[\Delta_{3/2}] + \left(\frac{57\sqrt{10}+10\sqrt{15}}{90} c_3 - \frac{1}{\sqrt{2}} s_3 \right) A_{3/2}^N[\Delta_{3/2}] = 0 \\
\\
& \frac{1}{3\sqrt{10}} (8\sqrt{2} c_1 + s_1) A_{1/2}^n[N_{1/2}] + \frac{\sqrt{2}}{3} (2\sqrt{5} c_3 + s_3) A_{1/2}^n[N_{3/2}] + \frac{1}{\sqrt{2}} A_{1/2}^n[N_{5/2}] \\
& + A_{3/2}^n[N_{5/2}] + \frac{1}{3\sqrt{10}} (c_1 - 8\sqrt{2} s_1) A_{1/2}^n[N'_{1/2}] + \frac{\sqrt{2}}{3} (c_3 - 2\sqrt{5} s_3) A_{1/2}^n[N'_{3/2}] \\
& + \frac{10(3\sqrt{2}-2\sqrt{3})}{108\sqrt{5}} A_{1/2}^N[\Delta_{1/2}] + \frac{-663+5\sqrt{6}}{108\sqrt{5}} A_{1/2}^N[\Delta_{3/2}] + \frac{3(5\sqrt{2}+139\sqrt{3})}{108\sqrt{5}} A_{3/2}^N[\Delta_{3/2}] = 0
\end{aligned} \tag{21}$$

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- [1] J. Beringer *et al.* [Particle Data Group Collaboration], Phys. Rev. D **86**, 010001 (2012).
[2] F. E. Close, *An Introduction to Quarks and Partons* (Academic Press, London, 1979).
[3] C. E. Carlson, C. D. Carone, J. L. Goity and R. F. Lebed, Phys. Lett. B **438**, 327 (1998) [hep-ph/9807334].
[4] C. E. Carlson, C. D. Carone, J. L. Goity and R. F. Lebed, Phys. Rev. D **59**, 114008 (1999) [hep-ph/9812440].
[5] J. L. Goity, C. Schat and N. Scoccola, Phys. Rev. D **71**, 034016 (2005) [hep-ph/0411092].
[6] N. N. Scoccola, J. L. Goity and N. Matagne, Phys. Lett. B **663**, 222 (2008) [arXiv:0711.4203 [hep-ph]].
[7] G. t'Hooft, Nucl. Phys. **B72**, 461 (1974); E. Witten, Nucl. Phys. **B160**, 57 (1979).
[8] J. -L. Gervais and B. Sakita, Phys. Rev. Lett. **52**, 87 (1984).
[9] J. -L. Gervais and B. Sakita, Phys. Rev. D **30**, 1795 (1984).
[10] R. F. Dashen and A. V. Manohar, Phys. Lett. B **315**, 425 (1993) [hep-ph/9307241].

- [11] R. F. Dashen and A. V. Manohar, Phys. Lett. B **315**, 438 (1993) [hep-ph/9307242].
- [12] C.D. Carone, H. Georgi, L. Kaplan, and D. Morin, Phys. Rev. **D50** 5793 (1994).
- [13] C. E. Carlson and C. D. Carone, Phys. Rev. **D58** 053005 (1998); Phys. Lett. **B441** 363 (1998).
- [14] C. L. Schat, J. L. Goity and N. N. Scoccola, Phys. Rev. Lett. **88**, 102002 (2002); J. L. Goity, C. L. Schat and N. N. Scoccola, Phys. Rev. D **66**, 114014 (2002).
- [15] T. D. Cohen and R. F. Lebed, Phys. Rev. Lett. **91**, 012001 (2003); Phys. Rev. D **67**, 096008 (2003); Phys. Rev. D **68**, 056003 (2003); T. D. Cohen, D. C. Dakin, A. Nellore and R. F. Lebed, Phys. Rev. D **69**, 056001 (2004); T. D. Cohen, D. C. Dakin, R. F. Lebed and D. R. Martin, Phys. Rev. D **71**, 076010 (2005); T. D. Cohen and R. F. Lebed, Phys. Lett. B **619**, 115 (2005); Phys. Rev. D **72**, 056001 (2005).
- [16] N. Matagne and F. Stancu, Phys. Rev. **D 71**, 014010 (2005); Phys. Lett. **B 631**, 7 (2005); Phys. Rev. D **73**, 114025 (2006); Phys. Rev. D **84**, 056013 (2011); Phys. Rev. D **85**, 116003 (2012); Phys. Rev. D **86**, 076007 (2012).
- [17] J. L. Goity, C. Schat and N. N. Scoccola, Phys. Lett. B **564**, 83 (2003); J. L. Goity and N. N. Scoccola, Phys. Rev. D **72**, 034024 (2005); Phys. Rev. Lett. **99**, 062002 (2007); J. L. Goity, C. Jayalath and N. N. Scoccola, Phys. Rev. D **80**, 074027 (2009); C. Jayalath, J. L. Goity, E. Gonzalez de Urreta and N. N. Scoccola, Phys. Rev. D **84**, 074012 (2011).
- [18] B. Lucini and M. Panero, Phys. Rept. **526**, 93 (2013) [arXiv:1210.4997 [hep-th]].
- [19] J. L. Goity, Phys. Atom. Nucl. **68**, 624 (2005) [Yad. Fiz. **68**, 655 (2005)] [hep-ph/0405304].
- [20] R. Koniuk and N. Isgur, Phys. Rev. D **21**, 1868 (1980) [Erratum-ibid. D **23**, 818 (1981)].
- [21] D. Pirjol and C. Schat, Phys. Rev. D **67**, 096009 (2003) [hep-ph/0301187].
- [22] A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, New Jersey, 1974).
- [23] R. F. Dashen, E. E. Jenkins and A. V. Manohar, Phys. Rev. D **51**, 3697 (1995) [hep-ph/9411234].
- [24] J. L. Goity, Phys. Lett. B **414**, 140 (1997) [hep-ph/9612252].
- [25] A. V. Manohar, *At the frontier of particle physics*, vol. 1, Shifman, M. ed. (World Scientific, Singapore, 2001) p. 507.
- [26] N. Isgur and G. Karl, Phys. Lett. B **72**, 109 (1977).
- [27] N. Isgur and G. Karl, Phys. Rev. D **18**, 4187 (1978).
- [28] R. G. Edwards, N. Mathur, D. G. Richards and S. J. Wallace, Phys. Rev. D **87**, 054506 (2013)

[arXiv:1212.5236 [hep-ph]].

[29] R. G. Moorhouse, Phys. Rev. Lett. **16** 772 (1966).

[30] D. Faiman and D. E. Plane, Nucl. Phys. **B 50**, 379 (1972); A. J. G. Hey, P. J. Litchfield and R. J. Cashmore, Nucl. Phys. **B 95**, 516 (1975).

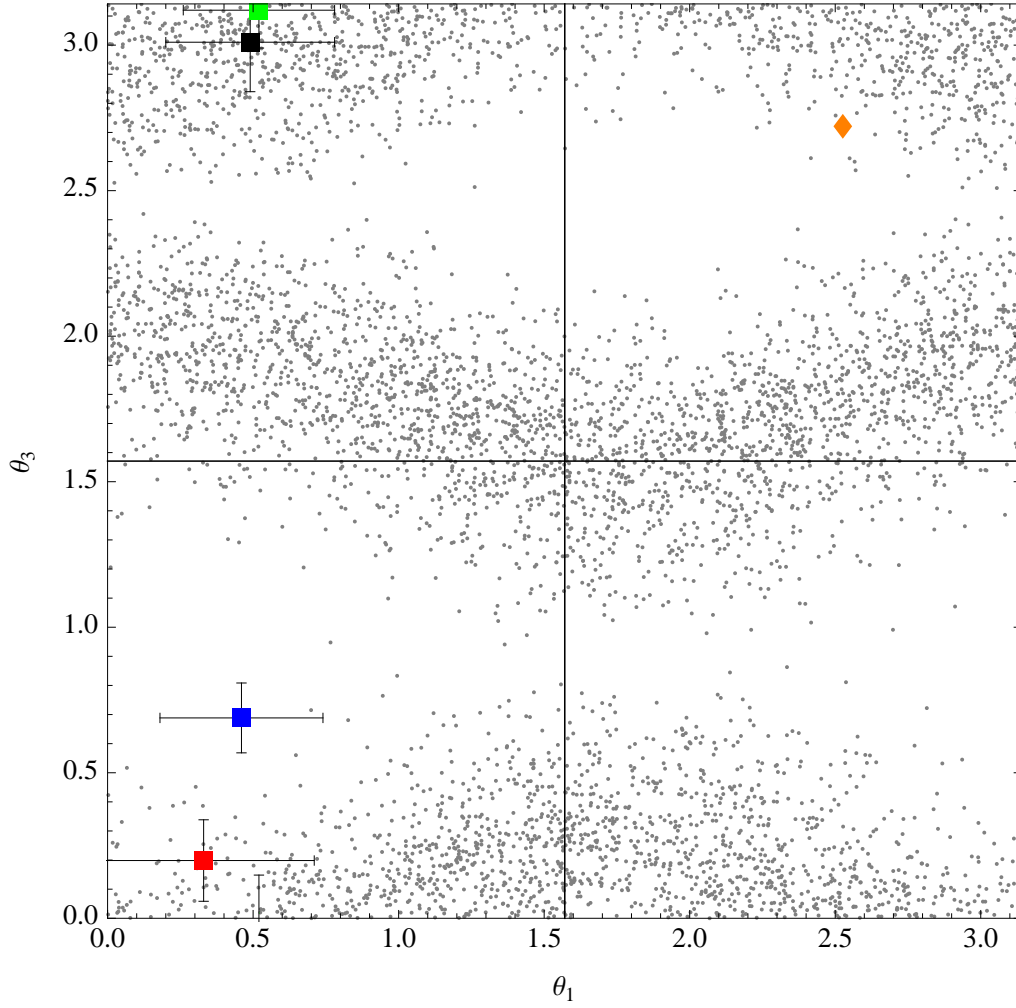


FIG. 1: Distribution of possible values of the angles θ_1 and θ_3 : The $\mathcal{O}(N_c^0)$ angles are depicted by the orange diamond; angles consistent with Eq. (15) using the empirical masses and the assignment of states defined in section [II] as Set 1 are depicted by the gray distribution of points (for other assignments shift the angles by $\pi/2$ accordingly), and the angles from the global fits obtained in this work are depicted in black, red, blue, and green respectively for assignments Set 1 through 4.

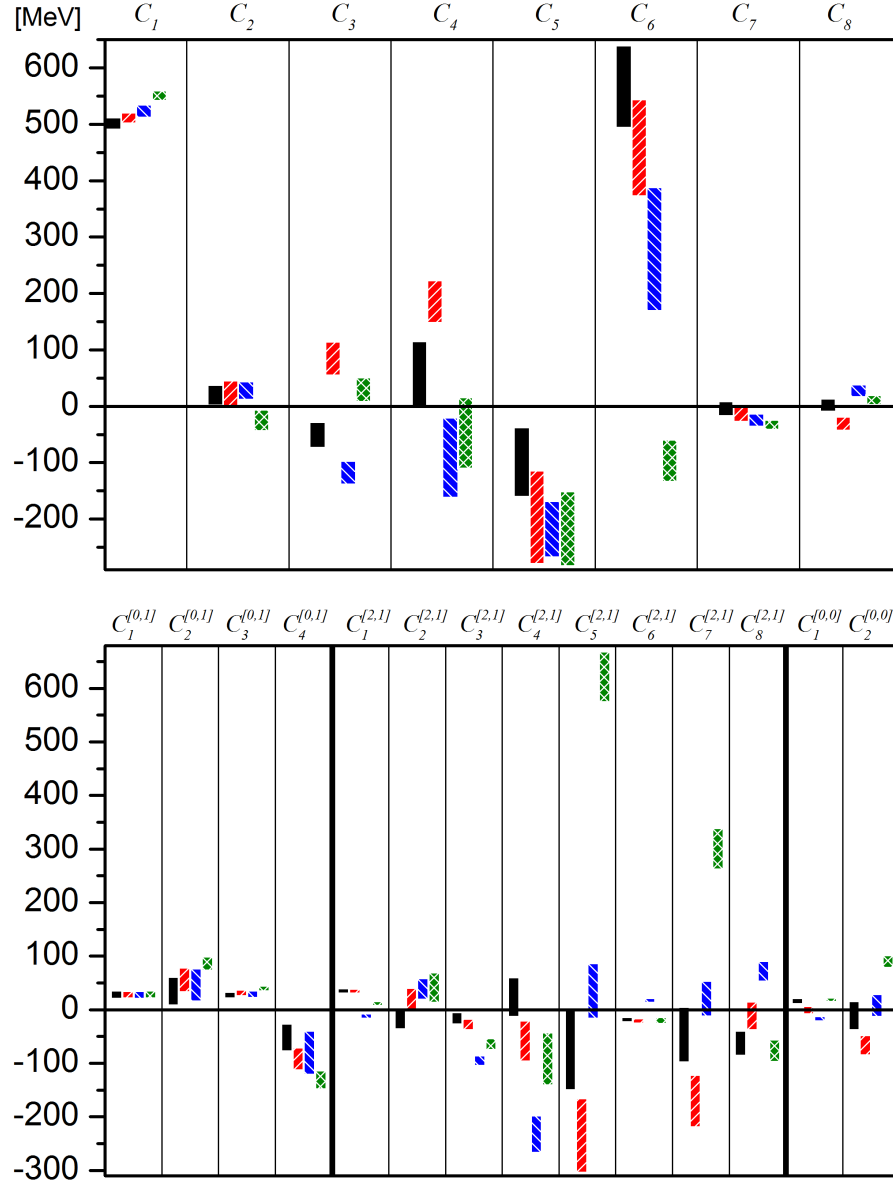


FIG. 2: Coefficients with errors from the global fits for the different assignments of states: black, red, blue and green respectively for Sets 1 through 4.

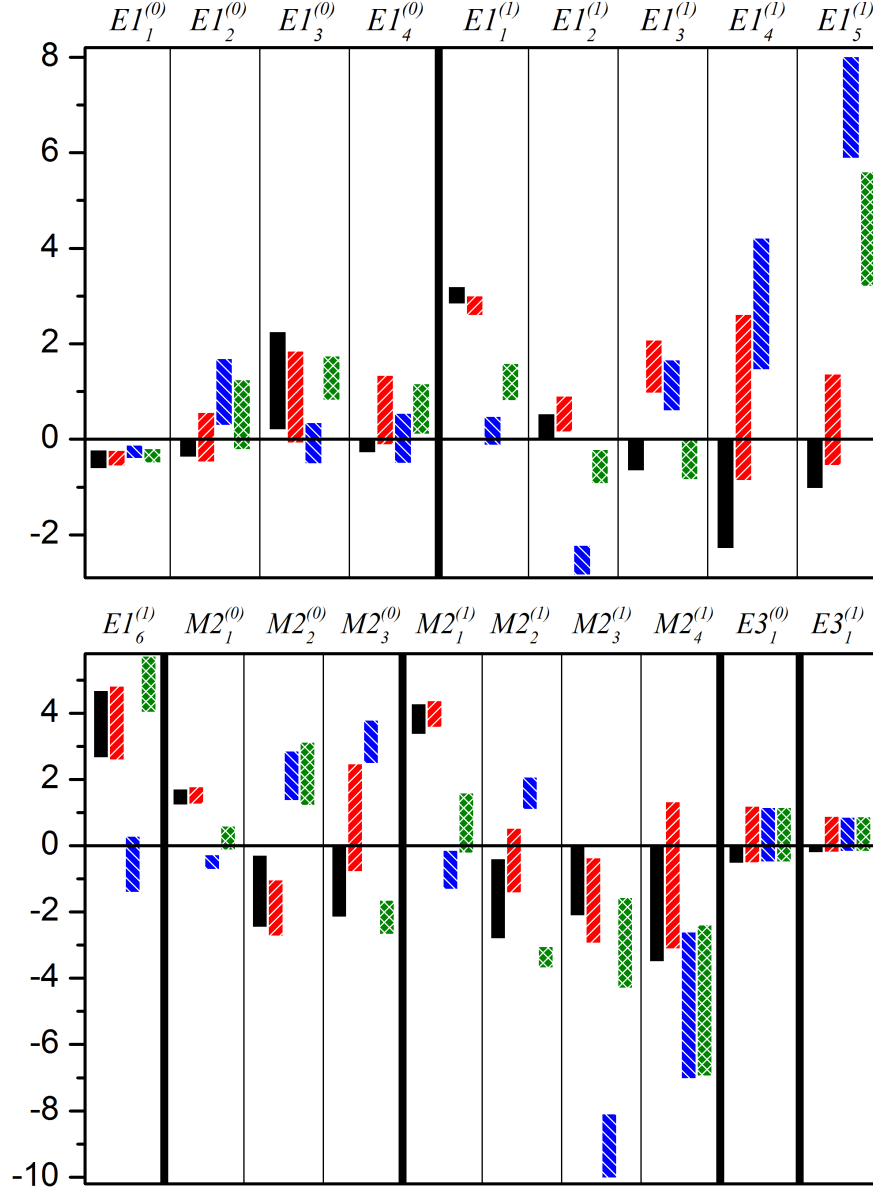


FIG. 3: Coefficients with errors from the global fits for the different assignments of states: black, red, blue and green respectively for Sets 1 through 4.

State	Mass	Width	Branching ratio [%]		$A_{\frac{1}{2}}^N$		$A_{\frac{3}{2}}^N$	
			S-wave	D-wave	$A_{\frac{1}{2}}^p$	$A_{\frac{1}{2}}^n$	$A_{\frac{3}{2}}^p$	$A_{\frac{3}{2}}^n$
	[MeV]	[MeV]	$[10^{-3} \times \text{GeV}^{-\frac{1}{2}}]$					
$N(1535)$	1538(18)	150(25)	$\pi N : 45(10)$	$\pi \Delta : 2(2)$	90(25)	-46(27)		
			$\eta N : 42(10)$					
$N(1520)$	1523(8)	113(13)	$\pi \Delta : 15(5)$	$\pi N : 60(5)$	-24(9)	-59(9)	166(5)	-139(11)
			$\pi \Delta : 13(3)$					
			$\eta N : \text{unknown}$					
$N(1650)$	1660(20)	150(30)	$\pi N : 70(20)$	$\pi \Delta : 13(13)$	53(16)	-15(21)		
			$\eta N : 10(5)$					
$N(1700)$	1700(50)	175(75)	$\pi \Delta : 50(40)$	$\pi N : 12(5)$	-18(13)	0(50)	-2(24)	-3(44)
			$\pi \Delta : 10(10)$					
			$\eta N : \text{unknown}$					
$N(1675)$	1678(8)	148(18)		$\pi N : 40(5)$	19(8)	-43(12)	15(9)	-58(13)
				$\pi \Delta : 50(15)$				
			$\eta N : \text{unknown}$					
$\Delta(1620)$	1645(30)	143(8)	$\pi N : 25(5)$	$\pi \Delta : 45(15)$	27(11)			
			$\eta \Delta : \text{no phase space}$					
$\Delta(1700)$	1720(50)	300(100)	$\pi \Delta : 38(13)$	$\pi N : 15(5)$	104(15)		85(22)	
			$\eta \Delta : \text{no phase space}$	$\pi \Delta : 10(5)$				
				$\eta \Delta : \text{no phase space}$				

TABLE I: Non-strange negative parity baryons with their partial decay widths and helicity photoproduction amplitudes as given by the PDG [1].

Set		1	2	3	4
Naturalness	M	✓	✓	✗	✓
	D	✓	✓	✗	✗
	EM	✓	✓	✓	✗
1-B dominance	D	✓	✗	✗	✗
	EM	✓	✓	✗	✗
HF term	M	✓	✓	✓	✗

TABLE II: Summary of application of the criteria discussed in the text to results shown in Figs.

2 and 3. M , D and EM refer to masses, decays and EM helicity amplitudes respectively.

Mass [MeV]			Strong decays			EM helicity amplitudes		
	Fit 1	Fit 2		Fit 1	Fit 2		Fit 1	Fit 2
C_1	497(5)	497(3)	$C_1^{[0,1]}$	23(2)	22(2)	$E1_1^{(0)}$	-0.4(0.2)	-0.4(0.2)
C_2	20(17)	28(11)	$C_2^{[0,1]}$	35(27)	25(12)	$E1_2^{(0)}$	0.2(0.6)	-
C_3	-50(22)	-48(10)	$C_3^{[0,1]}$	27(6)	26(5)	$E1_3^{(0)}$	1(1)	0.7(0.7)
C_4	57(58)	32(27)	$C_4^{[0,1]}$	-52(26)	-41(16)	$E1_4^{(0)}$	0.4(0.7)	-
C_5	-99(61)	-98(32)	$C_1^{[2,1]}$	3.6(0.2)	3.6(0.2)	$E1_1^{(1)}$	3.0(0.2)	3.1(0.2)
C_6	567(72)	577(41)	$C_2^{[2,1]}$	-2(2)	-	$E1_2^{(1)}$	0.3(0.3)	0.3(0.2)
C_7	-4(13)	-	$C_3^{[2,1]}$	-2(1)	-2(1)	$E1_3^{(1)}$	-0.1(0.6)	-
C_8	3(11)	-	$C_4^{[2,1]}$	2(4)	-	$E1_4^{(1)}$	0(2)	-
Mixing angles			$C_5^{[2,1]}$	-8(8)	-	$E1_5^{(1)}$	-0.1(0.9)	-
			$C_6^{[2,1]}$	-1.6(0.2)	-1.5(0.2)	$E1_6^{(1)}$	4(1)	3.8(0.7)
	Fit 1	Fit 2	$C_7^{[2,1]}$	-5(5)	-	$M2_1^{(0)}$	1.5(0.3)	1.5(0.3)
θ_1	0.49(0.29)	0.40(0.13)	$C_8^{[2,1]}$	-6(2)	-7(2)	$M2_2^{(0)}$	-1(1)	-1.6(0.8)
θ_3	3.01(0.17)	2.96(0.05)	$C_1^{[0,0]}$	16(6)	18(2)	$M2_3^{(0)}$	0(2)	-
			$C_2^{[0,0]}$	-11(27)	-	$M2_1^{(1)}$	3.8(0.5)	3.8(0.4)
						$M2_2^{(1)}$	-1.6(1.2)	-2.2(0.6)
	Fit 1	Fit 2				$M2_3^{(1)}$	-1(1)	-
χ^2_{dof}	0.39	0.34				$M2_4^{(1)}$	-1(2)	-
dof	5	22				$E3_1^{(0)}$	0.3(0.9)	-
						$E3_1^{(1)}$	0.3(0.6)	-

TABLE III: Results for the fits to Set 1. The parameters fitted are those associated with the basis operators for masses, strong decays and helicity amplitudes. The parameters for the latter two are dimensionless. The results for two fits are given. In Fit 1 all operators are included while in Fit 2 operators with coefficients compatible with zero are disregarded.

Masses [MeV]			
State	Empirical	Fit 1	Fit 2
$N(1535)$	1538(18)	1539	1539
$N(1520)$	1523(8)	1523	1523
$N(1650)$	1660(20)	1659	1658
$N(1700)$	1700(50)	1697	1718
$N(1675)$	1678(8)	1678	1678
$\Delta(1620)$	1645(30)	1651	1646
$\Delta(1700)$	1720(50)	1703	1701

Strong partial decay widths [MeV]			
Channel	Empirical	Fit 1	Fit 2
$N(1535) \rightarrow \pi N$	68(19)	61.5	61.4
$N(1520) \rightarrow \pi \Delta$	17(6)	15.0	14.6
π $N(1650) \rightarrow \pi N$	105(37)	95.4	88.3
S-wave $N(1700) \rightarrow \pi \Delta$	88(79)	133	149
$\Delta(1620) \rightarrow \pi N$	35(7)	35.6	36.3
$\Delta(1700) \rightarrow \pi \Delta$	113(53)	137	128
$N(1535) \rightarrow \pi \Delta$	3(3)	2.0	2.02
$N(1520) \rightarrow \pi N$	68(9)	66.0	72.6
$N(1520) \rightarrow \pi \Delta$	14(3)	12.4	9.36
$N(1650) \rightarrow \pi \Delta$	19(19)	22.8	19.5
π $N(1700) \rightarrow \pi N$	21(13)	20.5	26.0
D-wave $N(1700) \rightarrow \pi \Delta$	18(19)	20.4	24.2
$N(1675) \rightarrow \pi N$	59(10)	59.7	57.7
$N(1675) \rightarrow \pi \Delta$	73(24)	82.0	58.2
$\Delta(1620) \rightarrow \pi \Delta$	64(22)	75.3	74.1
$\Delta(1700) \rightarrow \pi N$	45(15)	44.7	47.0
$\Delta(1700) \rightarrow \pi \Delta$	30(18)	33.5	30.2
η $N(1535) \rightarrow \eta N$	63(18)	63.0	65.3
S-wave $N(1650) \rightarrow \eta N$	15(8)	15.0	12.6

TABLE IV: Results for masses and partial decay widths from the fits to Set 1.

EM helicity amplitudes $[10^{-3} \times \text{GeV}^{-\frac{1}{2}}]$				
	Empirical	Fit 1	Fit 2	$\eta(B^*)$
$A_{\frac{1}{2}}^p[N(1535)]$	90(30)	90.0	92.8	1
$A_{\frac{1}{2}}^n[N(1535)]$	-46(27)	-46.0	-47.7	1
$A_{\frac{1}{2}}^p[N(1520)]$	-24(9)	-24.0	-26.1	-1
$A_{\frac{1}{2}}^n[N(1520)]$	-59(9)	-59.0	-57.5	-1
$A_{\frac{3}{2}}^p[N(1520)]$	150(15)	150	151	-1
$A_{\frac{3}{2}}^n[N(1520)]$	-139(11)	-139	-142	-1
$A_{\frac{1}{2}}^p[N(1650)]$	53(16)	53.0	43.0	-1
$A_{\frac{1}{2}}^n[N(1650)]$	-15(21)	-15.0	-17.5	-1
$A_{\frac{1}{2}}^p[N(1700)]$	-18(13)	-18.0	-17.2	1
$A_{\frac{1}{2}}^n[N(1700)]$	0(50)	0.0	30.9	1
$A_{\frac{3}{2}}^p[N(1700)]$	-2(24)	-2.0	2.5	1
$A_{\frac{3}{2}}^n[N(1700)]$	-3(44)	-3.1	14.2	1
$A_{\frac{1}{2}}^p[N(1675)]$	19(8)	19.0	13.0	-1
$A_{\frac{1}{2}}^n[N(1675)]$	-43(12)	-43.0	-40.0	-1
$A_{\frac{3}{2}}^p[N(1675)]$	15(9)	15.0	18.3	-1
$A_{\frac{3}{2}}^n[N(1675)]$	-58(13)	-58.0	-56.6	-1
$A_{\frac{1}{2}}^N[\Delta(1620)]$	27(11)	27.0	27.2	1
$A_{\frac{1}{2}}^N[\Delta(1700)]$	104(15)	104	96.7	1
$A_{\frac{3}{2}}^N[\Delta(1700)]$	85(22)	85.0	95.5	1

TABLE V: Results for EM helicity amplitudes from the fits to Set 1. The last column gives the sign of the corresponding strong amplitude needed in Eq. (18).

Excited proton	$(S^* = \frac{1}{2}, S^* = \frac{3}{2})$ content	Moorhouse rule
p(1535)	(0.90, 0.43)	not suppressed
p(1650)	(-0.43, 0.90)	✓
p(1520)	(0.99, -0.13)	not suppressed
p(1700)	(0.13, 0.99)	✓
p(1675)	(0.0, 1.0)	✓

TABLE VI: Test of the Moorhouse rule in the proton helicity amplitudes.

n -Body	Operator	Order in $1/N_c$
0	$M_1 = N_c \mathbf{I}$	-1
1	$M_2 = \frac{6}{5} \sqrt{6} (ls)^{[0,0]}$	
2	$M_3 = \frac{144}{5} \sqrt{6} \frac{1}{N_c} \left(l^{(2)} (gG_c)^{[2,0]} \right)^{[0,0]}$	0
2	$M_4 = 2 \sqrt{\frac{2}{3}} \left(- (ls)^{[0,0]} + \frac{12}{N_c+3} \left(l (tG_c)^{[1,0]} \right)^{[0,0]} \right)$	
2	$M_5 = \frac{9}{5} \frac{1}{N_c} (lS_c)^{[0,0]}$	
2	$M_6 = -\frac{9}{2\sqrt{2}} \frac{1}{N_c} (S_c S_c)^{[0,0]}$	
2	$M_7 = 3\sqrt{3} \frac{1}{N_c} (sS_c)^{[0,0]}$	
2	$M_8 = 6\sqrt{6} \frac{1}{N_c} \left(l^{(2)} (sS_c)^{[2,0]} \right)^{[0,0]}$	1

TABLE VII: Mass basis operators. Here the notation $l^{(2)} = (ll)^{[2,0]}$ is used.

Meson/Partial wave	n -Body	Operator	Order in $1/N_c$
π S-wave	1	$O_1^{[0,1]} = (\xi \ g)^{[0,1]}$	0
		$O_2^{[0,1]} = \frac{1}{N_c} \left(\xi \ (s \ T_c)^{[1,1]} \right)^{[0,1]}$	1
	2	$O_3^{[0,1]} = \frac{1}{N_c} \left(\xi \ (t \ S_c)^{[1,1]} \right)^{[0,1]}$	1
		$O_4^{[0,1]} = \frac{1}{N_c} \left(\xi \ (g \ S_c)^{[1,1]} \right)^{[0,1]}$	1
π D-wave	1	$O_1^{[2,1]} = (\xi \ g)^{[2,1]}$	0
		$O_2^{[2,1]} = \frac{1}{N_c} \left(\xi \ (s \ T_c)^{[1,1]} \right)^{[2,1]}$	1
		$O_3^{[2,1]} = \frac{1}{N_c} \left(\xi \ (t \ S_c)^{[1,1]} \right)^{[2,1]}$	1
	2	$O_4^{[2,1]} = \frac{1}{N_c} \left(\xi \ (g \ S_c)^{[1,1]} \right)^{[2,1]}$	1
		$O_5^{[2,1]} = \frac{1}{N_c} \left(\xi \ (g \ S_c)^{[2,1]} \right)^{[2,1]}$	1
		$O_6^{[2,1]} = \frac{1}{N_c} \left(\xi \ (s \ G_c)^{[2,1]} \right)^{[2,1]}$	0
	3	$O_7^{[2,1]} = \frac{1}{N_c^2} \left(\xi \ \left(s \ (\{S_c, G_c\})^{[2,1]} \right)^{[2,1]} \right)^{[2,1]}$	1
		$O_8^{[2,1]} = \frac{1}{N_c^2} \left(\xi \ \left(s \ (\{S_c, G_c\})^{[2,1]} \right)^{[3,1]} \right)^{[2,1]}$	1
η S-wave	1	$O_1^{[0,0]} = (\xi \ s)^{[0,0]}$	0
	2	$O_2^{[0,0]} = \frac{1}{N_c} \left(\xi \ (s \ S_c)^{[1,0]} \right)^{[0,0]}$	1
η D-wave	1	$O_1^{[2,0]} = (\xi \ s)^{[2,0]}$	0
	2	$O_2^{[2,0]} = \frac{1}{N_c} \left(\xi \ (s \ S_c)^{[1,0]} \right)^{[2,0]}$	1
		$O_3^{[2,0]} = \frac{1}{N_c} \left(\xi \ (s \ S_c)^{[2,0]} \right)^{[2,0]}$	1

TABLE VIII: Strong decay basis operators. The upper labels $^{[L,I]}$ denote angular momentum and isospin and how these are coupled. The vector ξ provides the transition from the $O(3)$ (orbital) $\ell = 1$ (excited baryon) to $\ell = 0$ (ground state baryon). It is defined by its reduced matrix element: $\langle \ell = 0 \mid \xi \mid \ell = 1 \rangle = \sqrt{3}$ [6].

Operator	Order in $1/N_c$	n -Body
$E1_1^{(0)} = (\xi s)^{[1,0]}$	0	1
$E1_2^{(0)} = \frac{1}{N_c} \left(\xi (s S_c)^{[0,0]} \right)^{[1,0]}$	1	2
$E1_3^{(0)} = \frac{1}{N_c} \left(\xi (s S_c)^{[1,0]} \right)^{[1,0]}$	1	2
$E1_4^{(0)} = \frac{1}{N_c} \left(\xi (s S_c)^{[2,0]} \right)^{[1,0]}$	1	2
$E1_1^{(1)} = (\xi t)^{[1,1]}$	0	1
$E1_2^{(1)} = (\xi g)^{[1,1]}$	0	1
$E1_3^{(1)} = \frac{1}{N_c} \left(\xi (s G_c)^{[2,1]} \right)^{[1,1]}$	0	2
$E1_4^{(1)} = \frac{1}{N_c} \left(\xi (s T_c)^{[1,1]} \right)^{[1,1]}$	1	2
$E1_5^{(1)} = \frac{1}{N_c} \left(\xi (s G_c)^{[0,1]} \right)^{[1,1]} + \frac{1}{4\sqrt{3}} E1_1^{(1)}$	1	2
$E1_6^{(1)} = \frac{1}{N_c} \left(\xi (s G_c)^{[1,1]} \right)^{[1,1]} + \frac{1}{2\sqrt{2}} E1_2^{(1)}$	1	2
$M2_1^{(0)} = (\xi s)^{[2,0]}$	0	1
$M2_2^{(0)} = \frac{1}{N_c} \left(\xi (s S_c)^{[1,0]} \right)^{[2,0]}$	1	2
$M2_3^{(0)} = \frac{1}{N_c} \left(\xi (s S_c)^{[2,0]} \right)^{[2,0]}$	1	2
$M2_1^{(1)} = (\xi g)^{[2,1]}$	0	1
$M2_2^{(1)} = \frac{1}{N_c} \left(\xi (s G_c)^{[2,1]} \right)^{[2,1]}$	0	2
$M2_3^{(1)} = \frac{1}{N_c} \left(\xi (s T_c)^{[1,1]} \right)^{[2,1]}$	1	2
$M2_4^{(1)} = \frac{1}{N_c} \left(\xi (s G_c)^{[1,1]} \right)^{[2,1]} + \frac{1}{2\sqrt{2}} M2_1^{(1)}$	1	2
$E3_1^{(0)} = \frac{1}{N_c} \left(\xi (s S_c)^{[2,0]} \right)^{[3,0]}$	1	2
$E3_1^{(1)} = \frac{1}{N_c} \left(\xi (s G_c)^{[2,1]} \right)^{[3,1]}$	0	2

TABLE IX: EM basis operators. The upper notation $X^{(0)}$ and $X^{(1)}$ indicates the isospin of the operator. The NLO operators $E1_5^{(1)}$, $E1_6^{(1)}$, and $M2_4^{(1)}$ involve linear combinations with LO operators in order to eliminate their projections onto the LO operators.